Cyprus Association of Civil Engineers (CYACE)

Neapolis University Pafos

POLITECNICO DI MILANO

ΣΠΟΛΜΗΚ

Online Seminar

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# Analysis and Design of Prestressed Concrete Structures


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POLITECNICO MILANO

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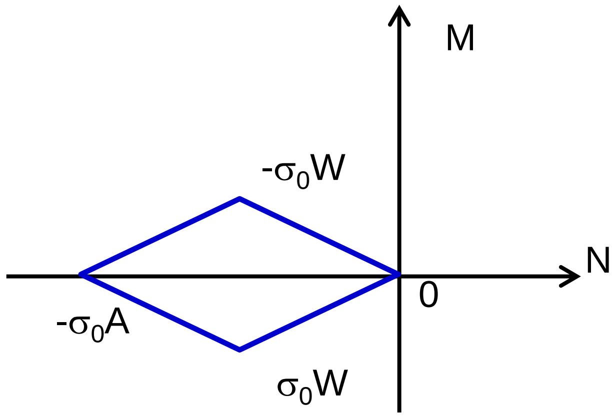
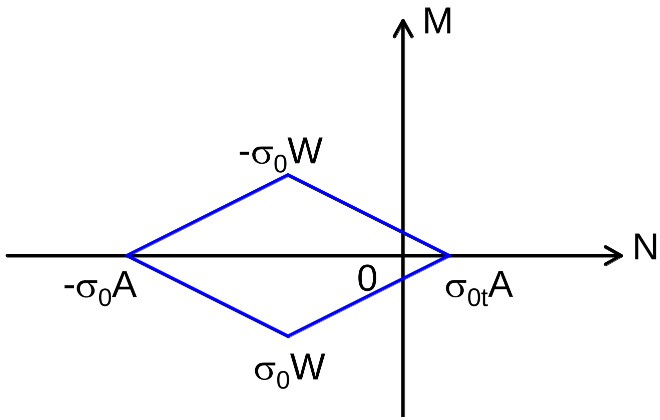
# PRESTRESSING: AN IDEA GENERATING AN OUTSANDING ENHANCEMENT IN STRUCTURAL ENGINEERING

# THE STONE MATERIALS AND THEIR INTRINSIC WEAKNESS



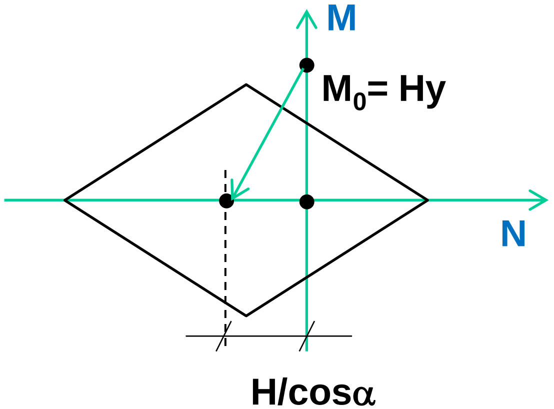

# THE ARCH AND THE RESISTANCE DUE TO THE SHAPE

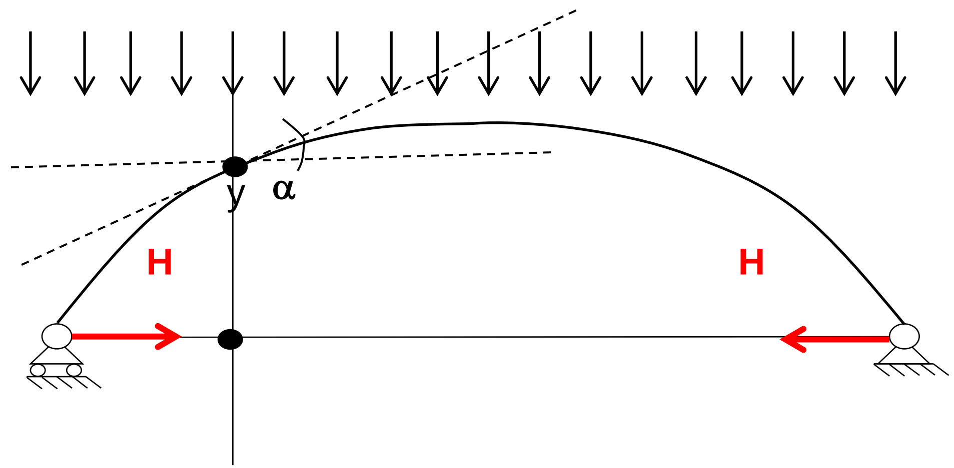
$M=M_0-Hy$

$M_0''=-q$

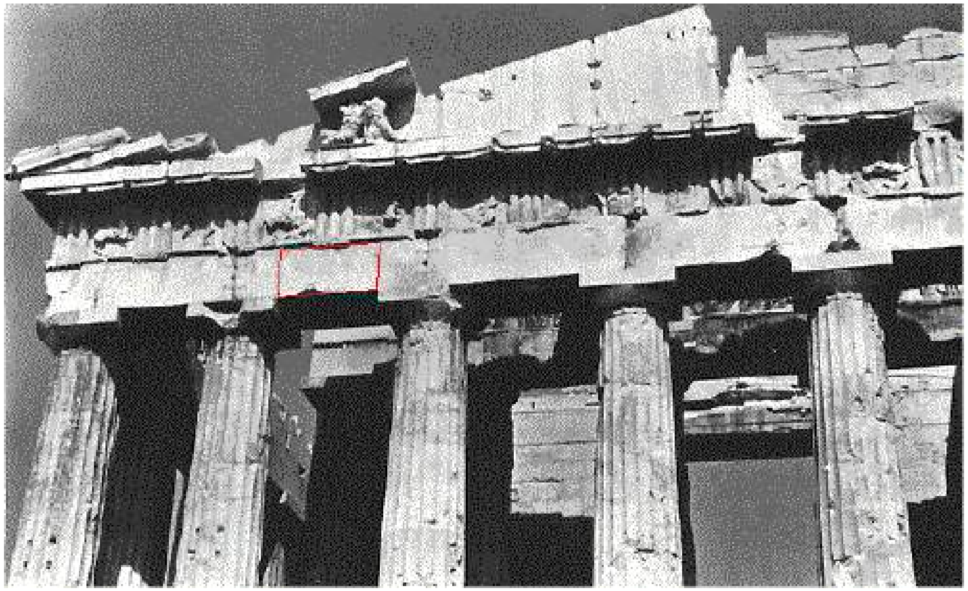
$M=0$ $\Rightarrow$ $Hy=M_0$

$Hy''=-q$

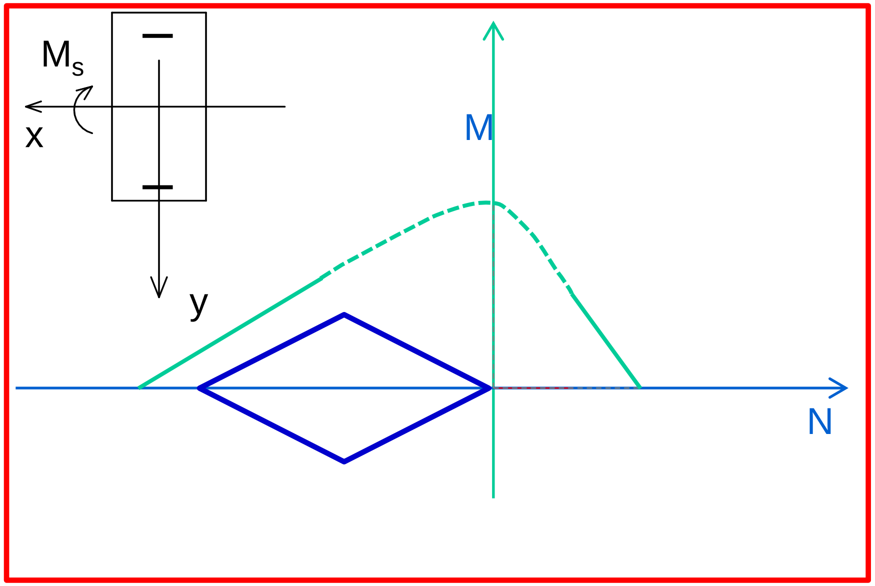


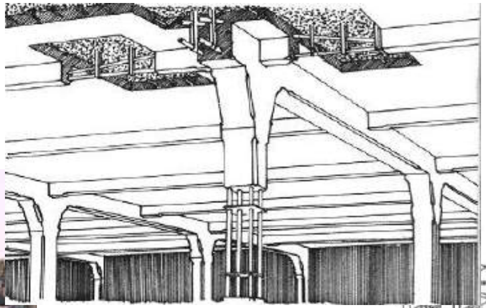



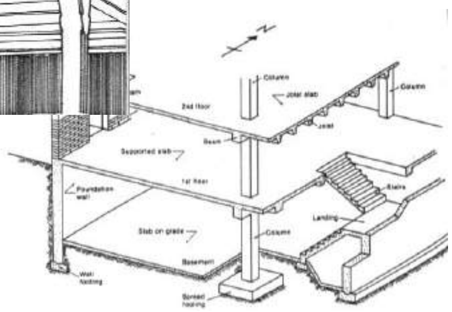
# THE STRUCTURAL FORMS AS A CONSEQUENCE OF THE MATERIAL BEHAVIOUR

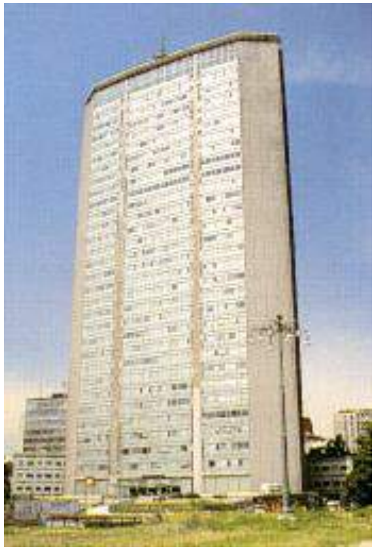


# A “ENHANCED” STONE. REINFORCED CONCRETE AS AN EXPRESSION OF THE STATICAL COLLABORATION OF NON-HOMOGENEOUS MATERIALS

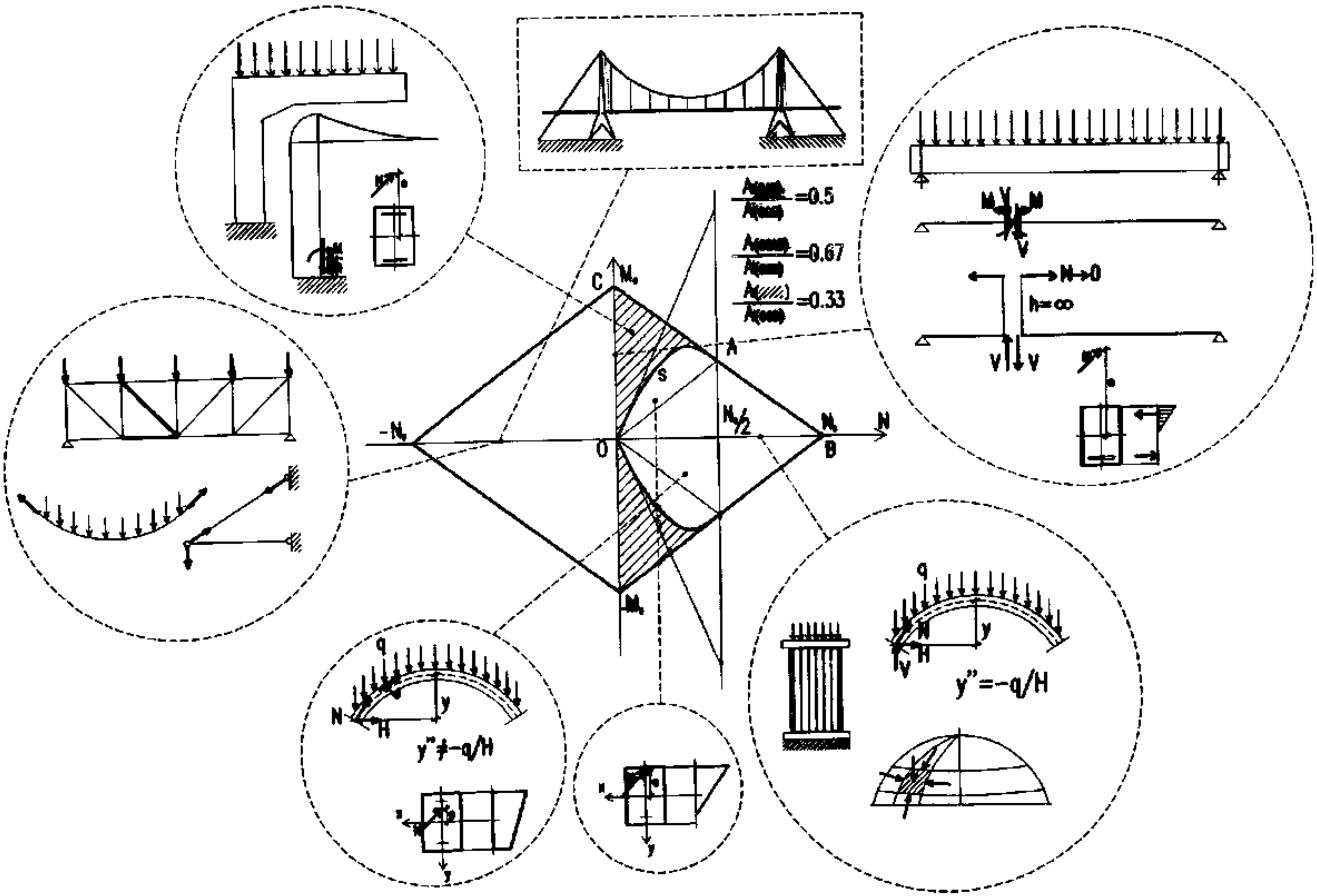






# NAVIER DIAGRAM AND STRUCTURAL BEHAVIOUR



# M-N DIAGRAM FOR A DOUBLY SYMMETRICAL SECTION

The coordinates of the points ABCD ($f_{ct} = |f_c|$):

$A_1$ ($N=f_{ct}A$; $M=0$)

$B_1$ ($N=0$; $M=\frac{W}{2}(f_{ct}-f_c)$)

$C_1$ ($N=f_cA$; $M=0$)

$D_1$ ($N=0$; $M=\frac{W}{2}(f_c-f_{ct})$)

$f_{ct} < |f_c|$

$A'_1$ ($N=f_{ct}A$; $M=0$)

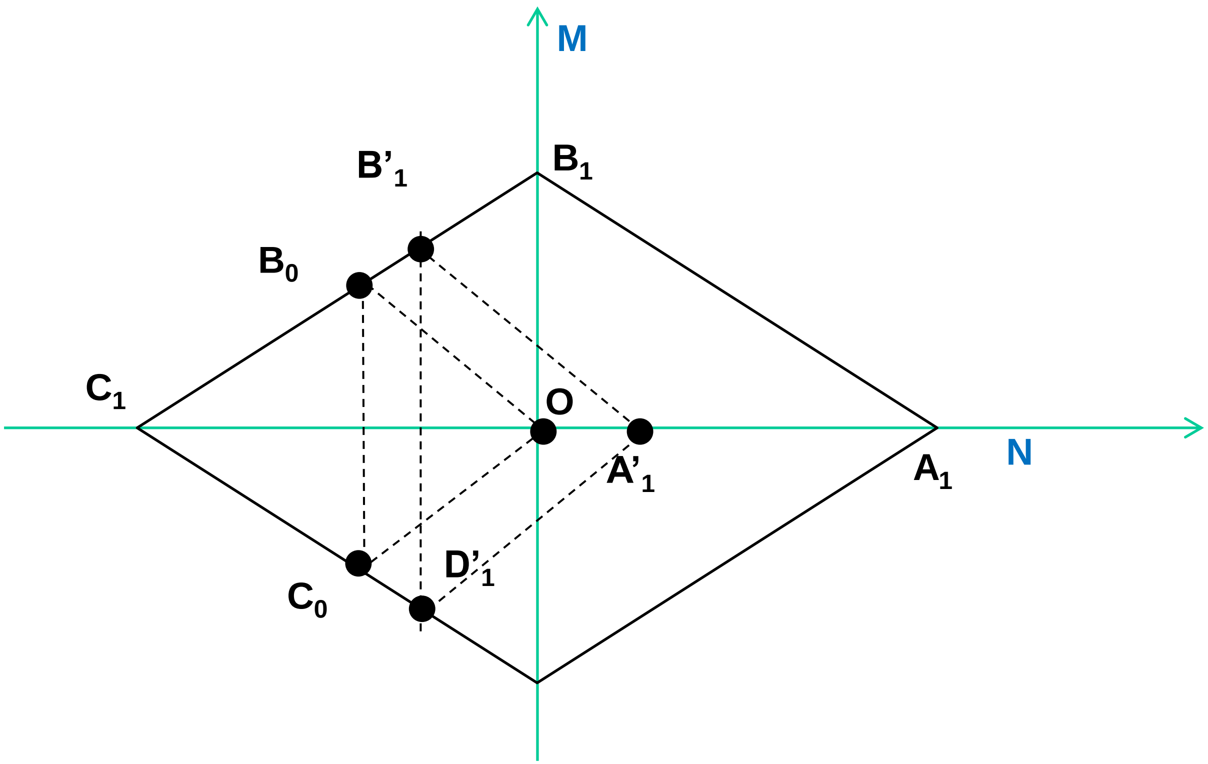
$B'_1$ ($N=\frac{A}{2}(f_{ct}+f_c)$; $M=\frac{W}{2}(f_{ct}-f_c)$)

$C'_1$ ($N=f_cA$; $M=0$)

$D'_1$ ($N=\frac{A}{2}(f_c+f_{ct})$; $M=\frac{W}{2}(f_c-f_{ct})$)

$f_{ct} = 0$

**$OB_0C_1C_0$**



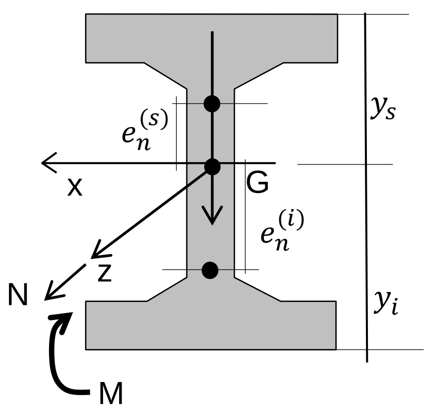
# SECTIONAL BEHAVIOUR IN THE SERVICE STAGE

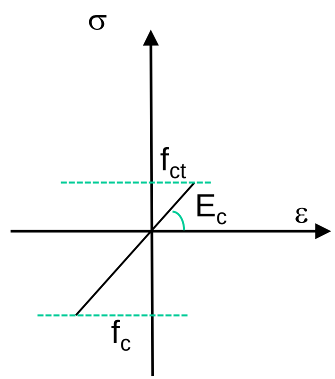
$$N_P = -Af_c \quad N_R = Af_{ct} \quad N_Q = (M_{ct} - M_c)/\eta h$$

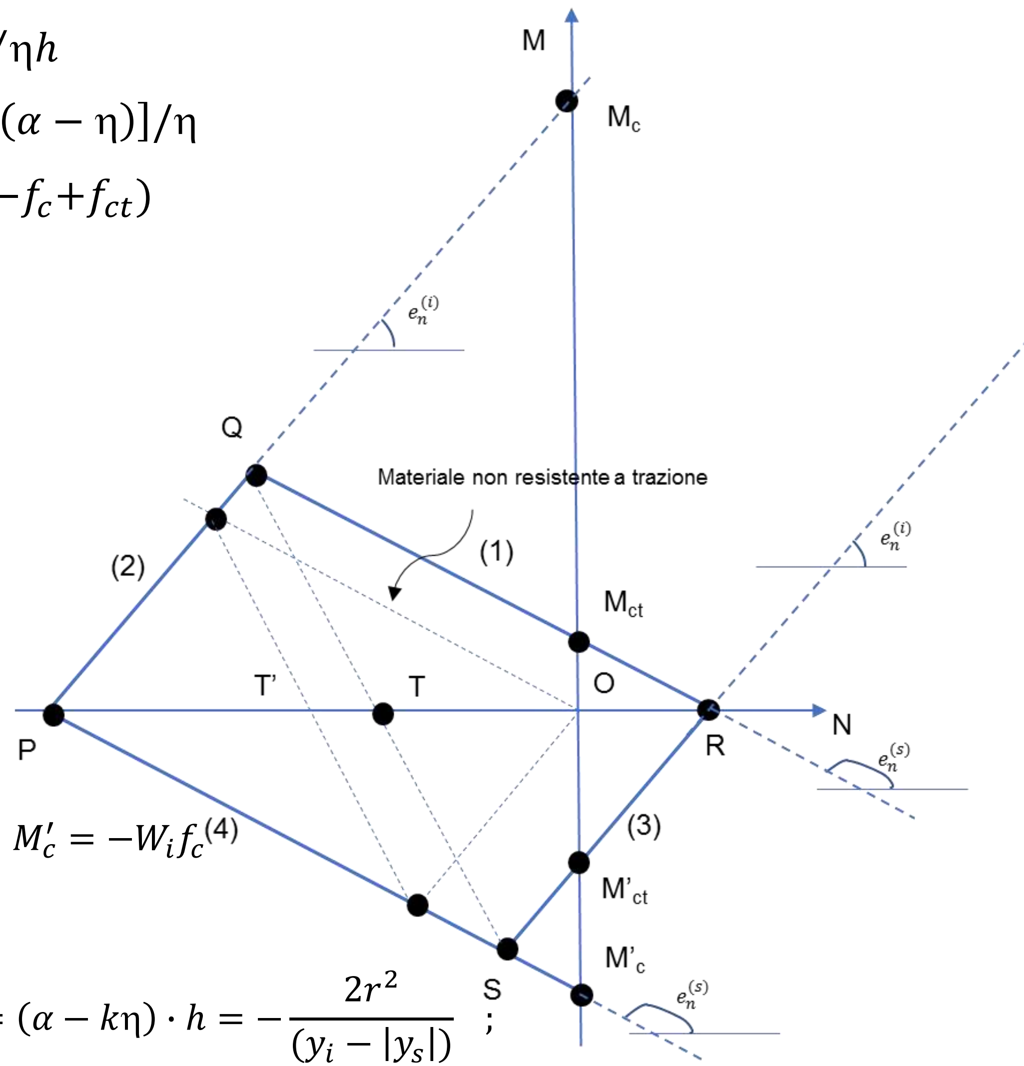
$$M_P = 0 \quad M_R = 0 \quad M_Q = [M_{ct}\alpha - M_c(\alpha - \eta)]/\eta$$

$$N_S = (M'_c - M'_{ct})/\eta h \qquad N_T = \frac{A}{2}(-f_c + f_{ct})$$

$$M_S = [-M'_{ct}(\alpha - \eta) + M'_c\alpha]/\eta \qquad M_T = 0$$




$$(1)\ \frac{N}{A} + \frac{M}{W_i} = -f_{ct} \qquad M_{ct} = W_i f_{ct}\ ;\quad M'_{ct} = -W_s f_{ct}\ ;\quad M'_c = -W_i f_c \ (4)$$

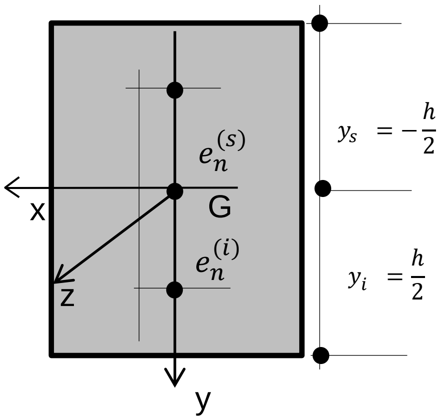
$$M_c = W_s f_{ct}$$

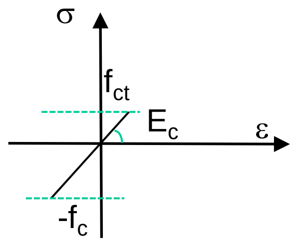
$$(2)\ \frac{N}{A} - \frac{M}{W_s} = -f_c$$

$$(3)\ \frac{N}{A} - \frac{M}{W_s} = f_{ct} \qquad \eta = \frac{r^2}{y_i|y_s|}\ ;\quad \alpha = \frac{e_n^{(i)}}{h}\ ;\quad tg\beta = (\alpha - k\eta)\cdot h = -\frac{2r^2}{(y_i - |y_s|)}\ ;$$

$$(4)\ \frac{N}{A} + \frac{M}{W_i} = -f_c \qquad k = \frac{(M_c - M'_{ct})}{(M_c - M'_{ct} + M'_c - M_{ct})}$$

# THE RECTANGULAR SECTION



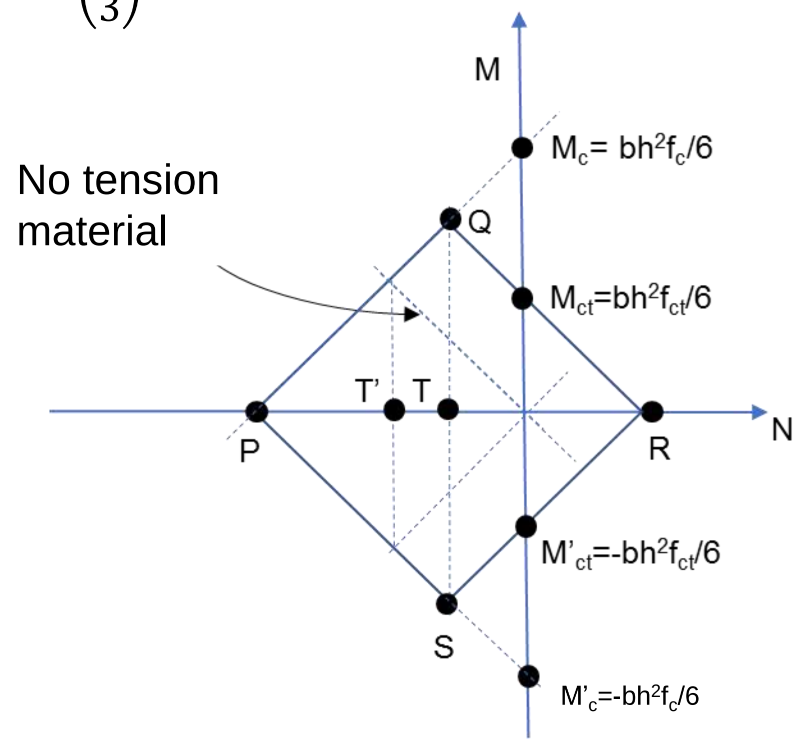




$$M_Q = \frac{\left[\frac{1}{6}M_{ct} + \frac{1}{6}M_c\right]}{\left(\frac{1}{3}\right)} = \frac{M_{ct} + M_c}{2}$$

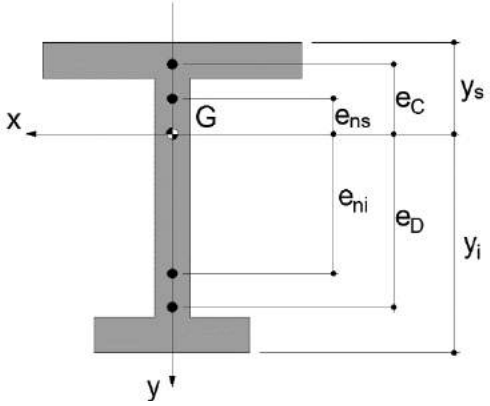
$$M_S = \frac{\left[\frac{1}{6}M'_{ct} + \frac{1}{6}M'_c\right]}{\left(\frac{1}{3}\right)} = \frac{M'_{ct} + M'_c}{2}$$

$$W_s = W_i = W = b\frac{h^2}{6} \ ; A_C = bh \ ; r^2 = \frac{h^2}{12}$$

$$y_i = |y_s| \ = \frac{h}{2} \ ; \eta = \frac{1}{3} \ ; \ \alpha = \frac{1}{6} \ ; \ \alpha - \eta = -\frac{1}{6}$$



# BASIC FEATURES OF THE M-N DIAGRAM




$$\begin{cases} N_A = Af_{ct} \\ M_A = 0 \\ e_A = 0 \end{cases} \quad \begin{cases} N_B = -Af_c \\ M_B = 0 \\ e_B = 0 \end{cases}$$

$$\begin{cases} N_C = \dfrac{A}{h}(-f_c y_i + f_{ct}|y_s|) \\ M_C = \dfrac{Ar^2}{h}(f_c + f_{ct}) \\ e_C = e_{ns}\left(1 - \dfrac{Af_{ct}}{N_c}\right) \end{cases}$$

$$\begin{cases} N_D = \dfrac{A}{h}(-f_c|y_s| + f_{ct}y_i) \\ M_D = -\dfrac{Ar^2}{h}(f_c + f_{ct}) \\ e_D = e_{ni}\left(1 - \dfrac{Af_{ct}}{N_D}\right) \end{cases}$$

Basic features of the M-N diagram:

$M_C = |M_D|$

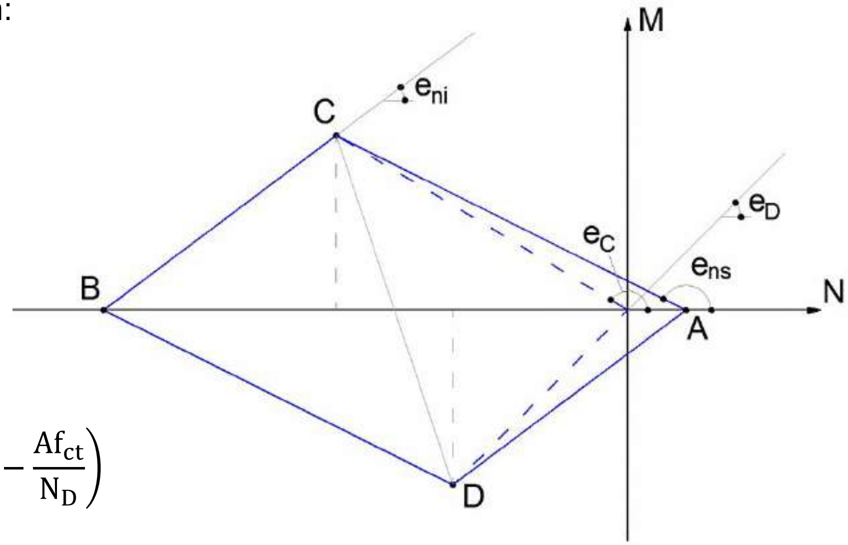
$N_D \geq N_C \quad$ for $y_i \geq |y_s|$

Kern eccentricities:

$$e_{ns} = -\frac{r^2}{y_i}; \quad e_{ni} = \frac{r^2}{|y_s|}$$

Limiting eccentricities:

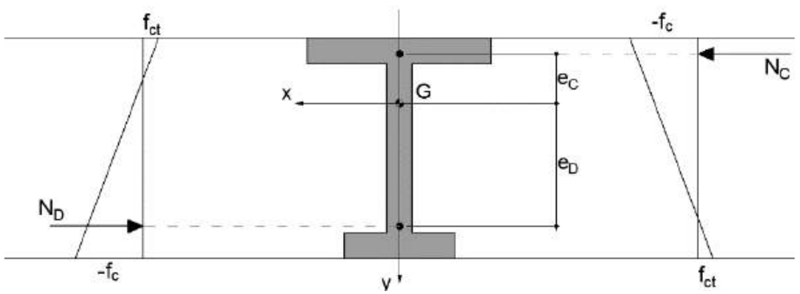
$$e_C = e_{ns}\left(1 - \frac{Af_{ct}}{N_c}\right); \quad e_D = e_{ni}\left(1 - \frac{Af_{ct}}{N_D}\right)$$




For doubly symmetric cross-sections:

$y_1 = |y_s| \qquad |e_{ns}| = e_{ni}$

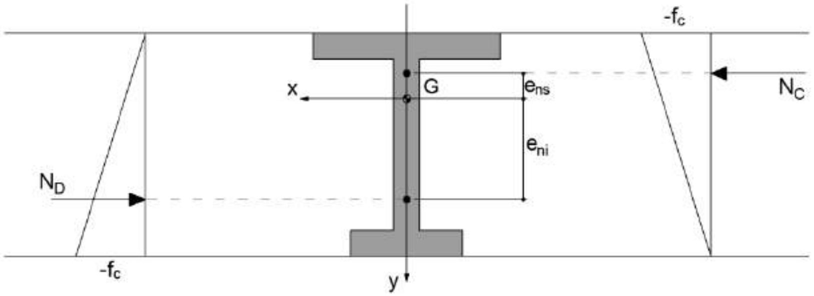
$N_C = N_D \qquad e_C = e_D$



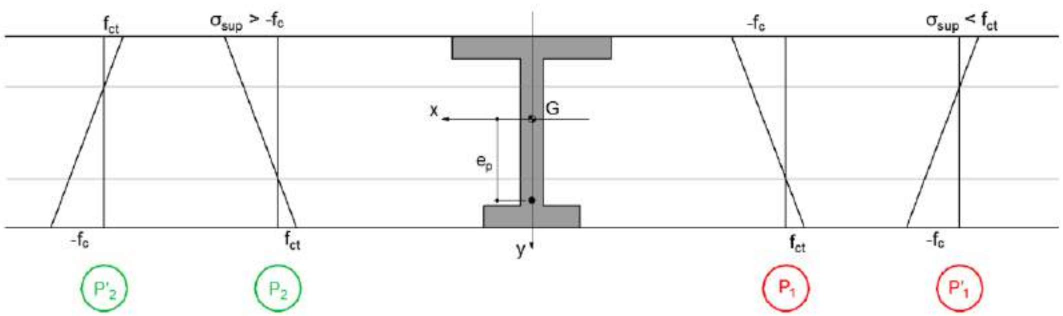

For no tension material:

$$N_D = N_C \frac{y_i}{|y_s|}$$

$e_C = e_{ns} \qquad e_D = e_{ni}$



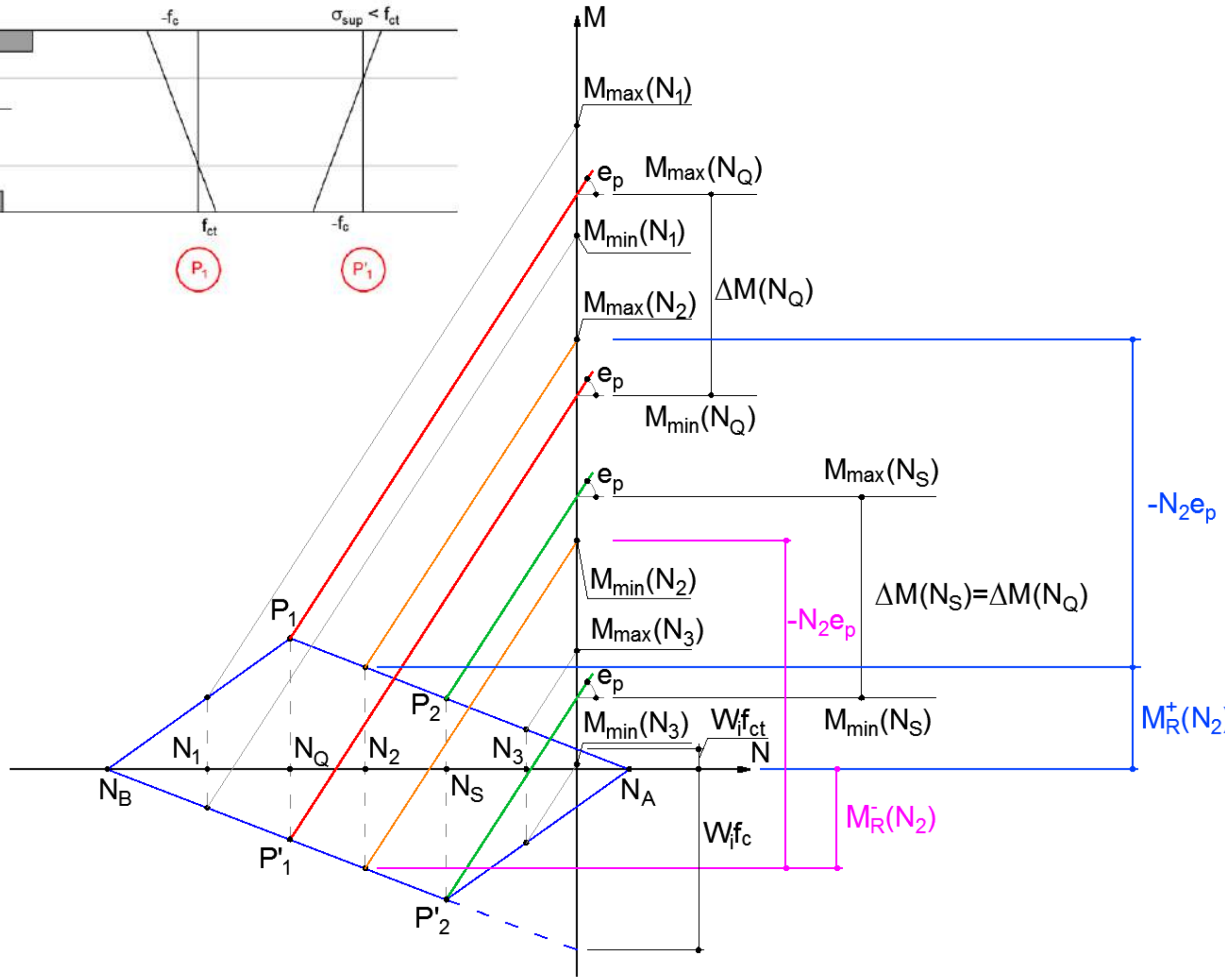
# THE BENDING SECTIONAL PERFORMANCE IN THE SERVICE STAGE




$$M_{max}(N) = M_R^+(N) - Ne_p \quad (1)$$

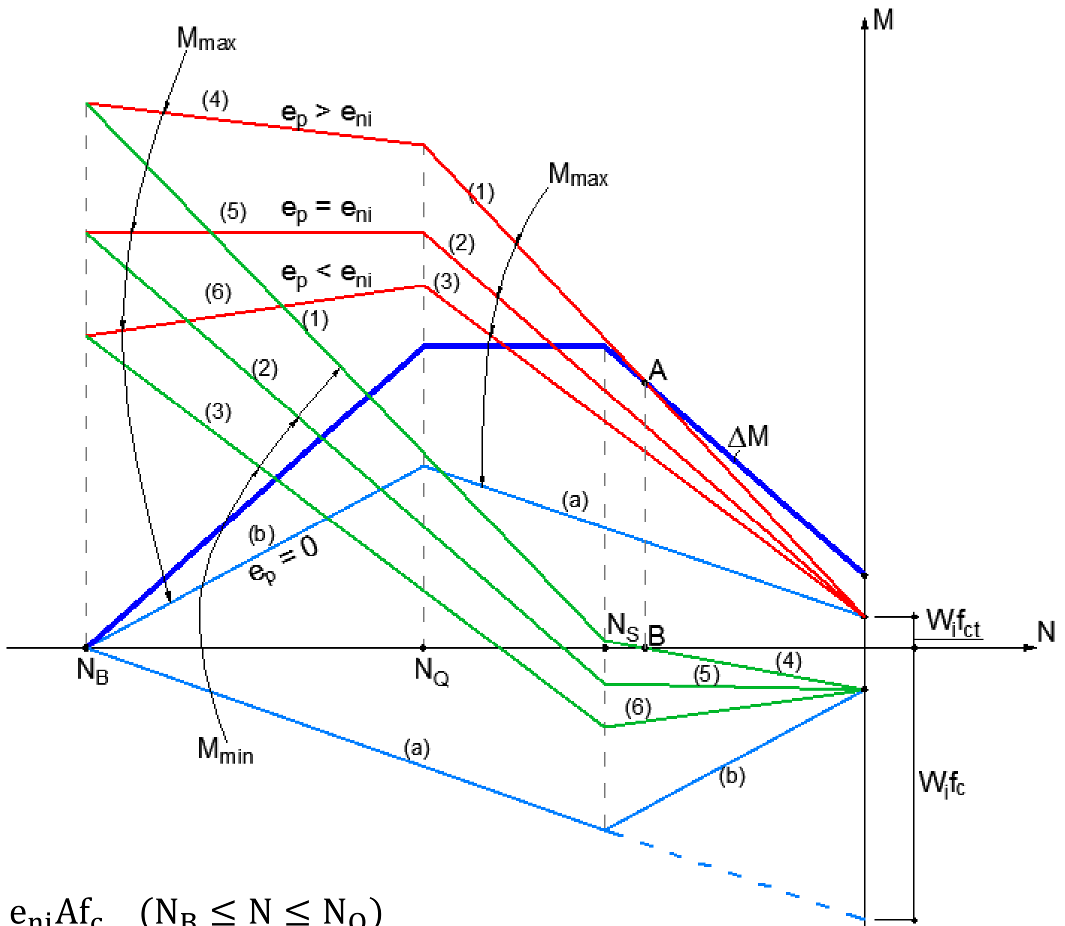
$$M_{min}(N) = M_R^-(N) - Ne_p \quad (2)$$

For a prescribed eccentricity $e_p$ eqs. (1), (2) represent the interaction diagrams connecting the maximum and minimum external moment to the applied axial load. In order to draw the interaction diagrams we have to analytically represent the sides of the parallelogram $AP_1BP'_2$ in terms of moment-axial load relationships.




$$\Delta M(N_Q) = \Delta M(N_S) = \Delta M_{max} = W_i(f_c + f_{ct})$$

# M-N AND ΔM-N INTERACTION DIAGRAMS




$$M_{max} = N(e_{ni} - e_p) + e_{ni}Af_c \quad (N_B \le N \le N_Q)$$

$$M_{max} = N(e_{ns} - e_p) - e_{ns}Af_{ct} \quad (N_Q \le N \le 0)$$

$$M_{min} = N(e_{ns} - e_p) + e_{ns}Af_c \quad (N_B \le N \le N_S)$$

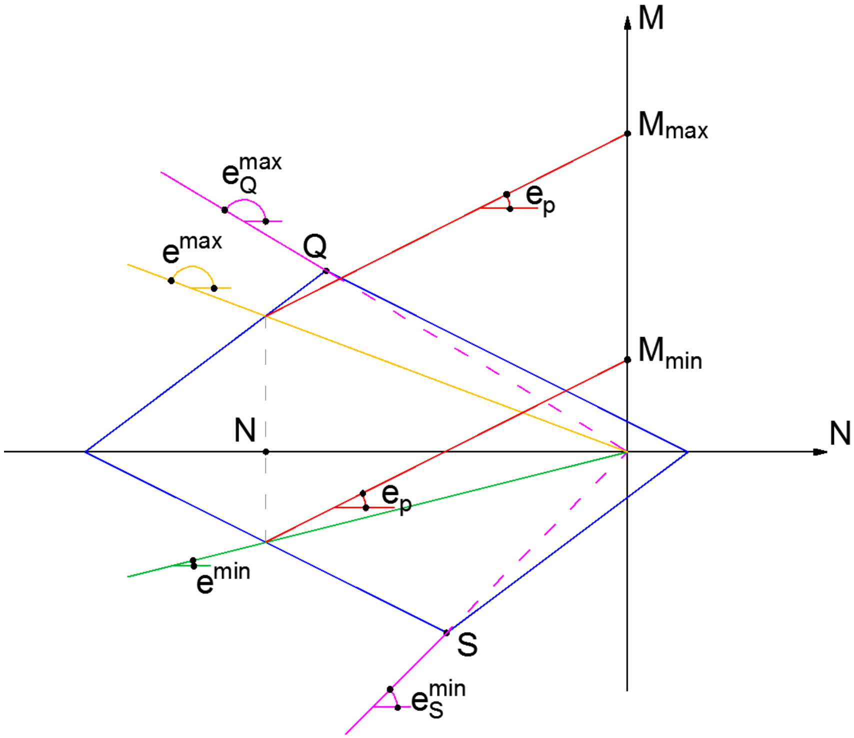
$$M_{min} = N(e_{ni} - e_p) - e_{ni}Af_{ct} \quad (N_S \le N \le 0)$$

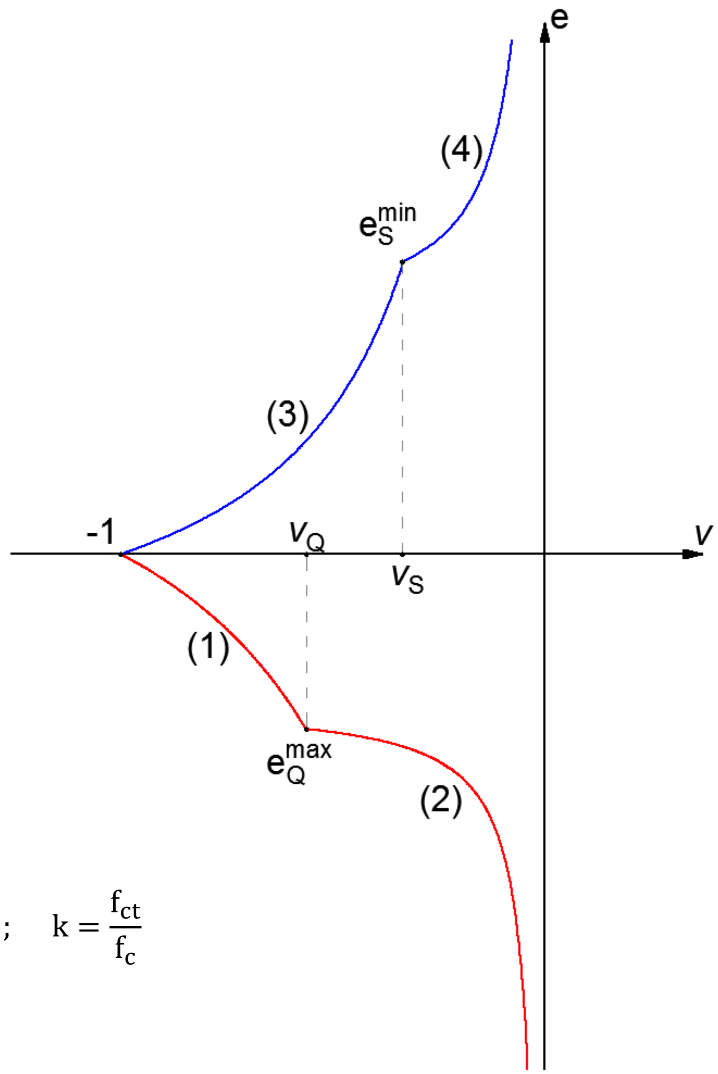
$$\Delta M = (e_{ni} - e_{ns})(N + Af_c) \quad (N_B \le N \le N_Q)$$

$$\Delta M = -e_{ns}(Af_{ct} + Af_c) = W_i(f_{ct} + f_c) \quad (N_Q \le N \le N_S)$$

$$\Delta M = (e_{ns} - e_{ni})(N - Af_{ct}) \quad (N_S \le N \le 0)$$

# e-ν DIAGRAM




(1) $e_{max} = e_{ni}\left(1 + \frac{1}{\nu}\right) \qquad -1 \le \nu \le \nu_Q$

(2) $e_{max} = e_{ns}\left(1 - \frac{k}{\nu}\right) \qquad \nu_Q \le \nu \le 0$

(3) $e_{min} = e_{ns}\left(1 + \frac{1}{\nu}\right) \qquad -1 \le \nu \le \nu_S$

(4) $e_{min} = e_{ni}\left(1 - \frac{k}{\nu}\right) \qquad \nu_S \le \nu \le 0$

$$\nu = \frac{Af_c}{N}; \qquad k = \frac{f_{ct}}{f_c}$$

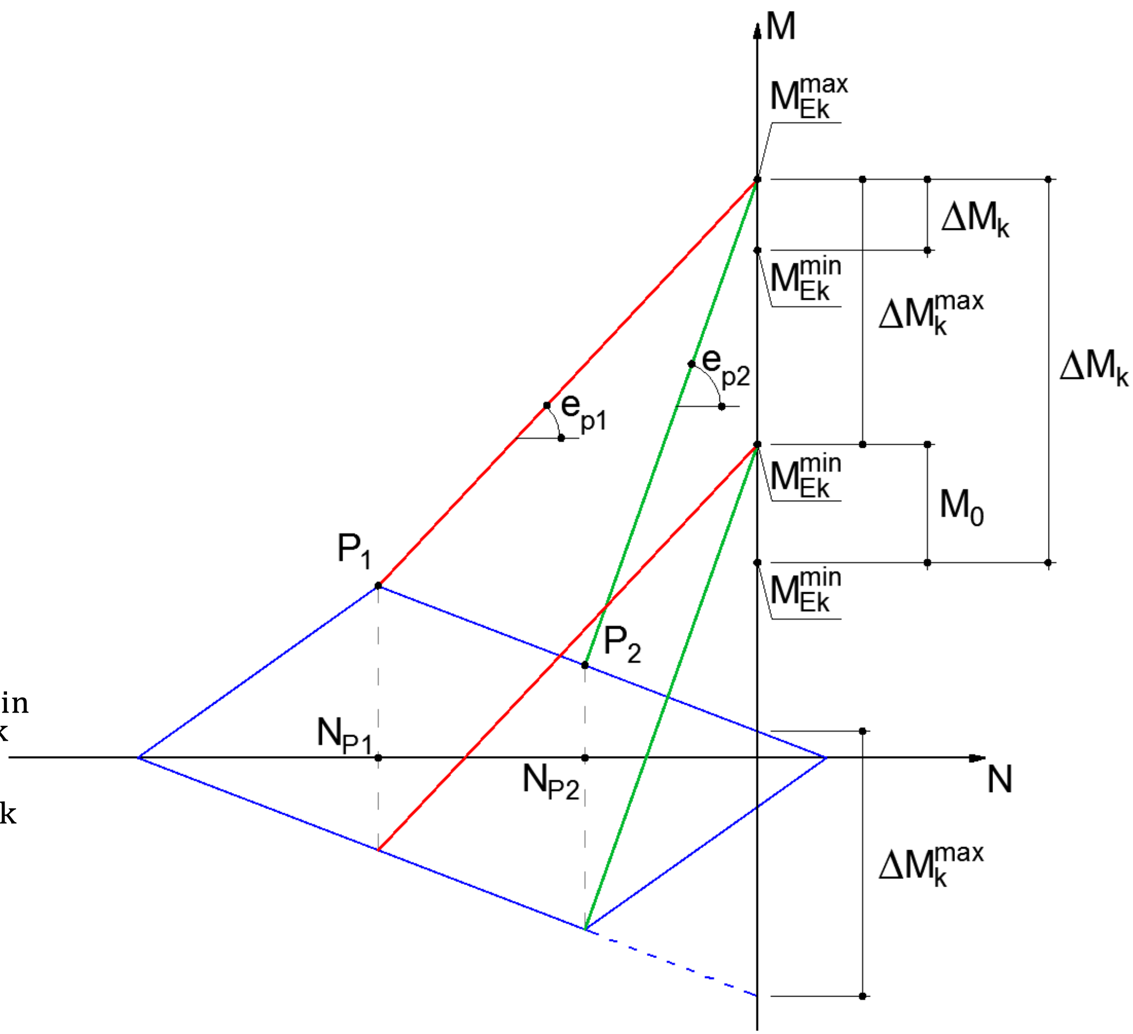
# THE DESIGN OF A P.C SECTION

If $\Delta M_k \leq \Delta M_k^{max}$

- $N_{P1} \leq N \leq N_{P2}$
- $e_{P1} \leq e \leq e_{P2}$

If $\Delta M_k \geq \Delta M_k^{max}$

- Increase $W_i$ , $f_c$
- Add a bending moment $M_0$ to $M_{Ek}^{min}$ so that $M_0 + M_{Ek}^{min} + \Delta M_k^{max} = M_{Ek}$




$$\Delta M_k^{max} = W_i(f_c + f_{ct})$$

Naming $M_0$, $N_0$ the resisting moment and axial load, we can write:

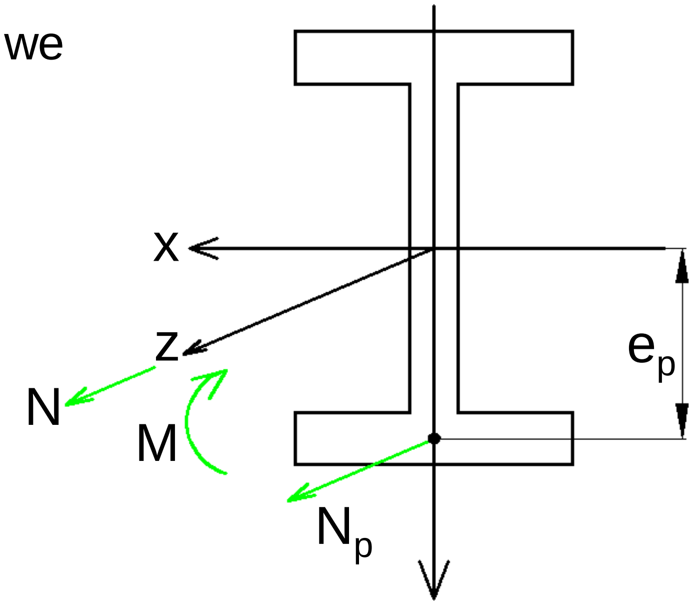
$$N_0 = N + N_p$$

$$M_0 = M + N_p e_p$$

The design action effects, M, N, become:

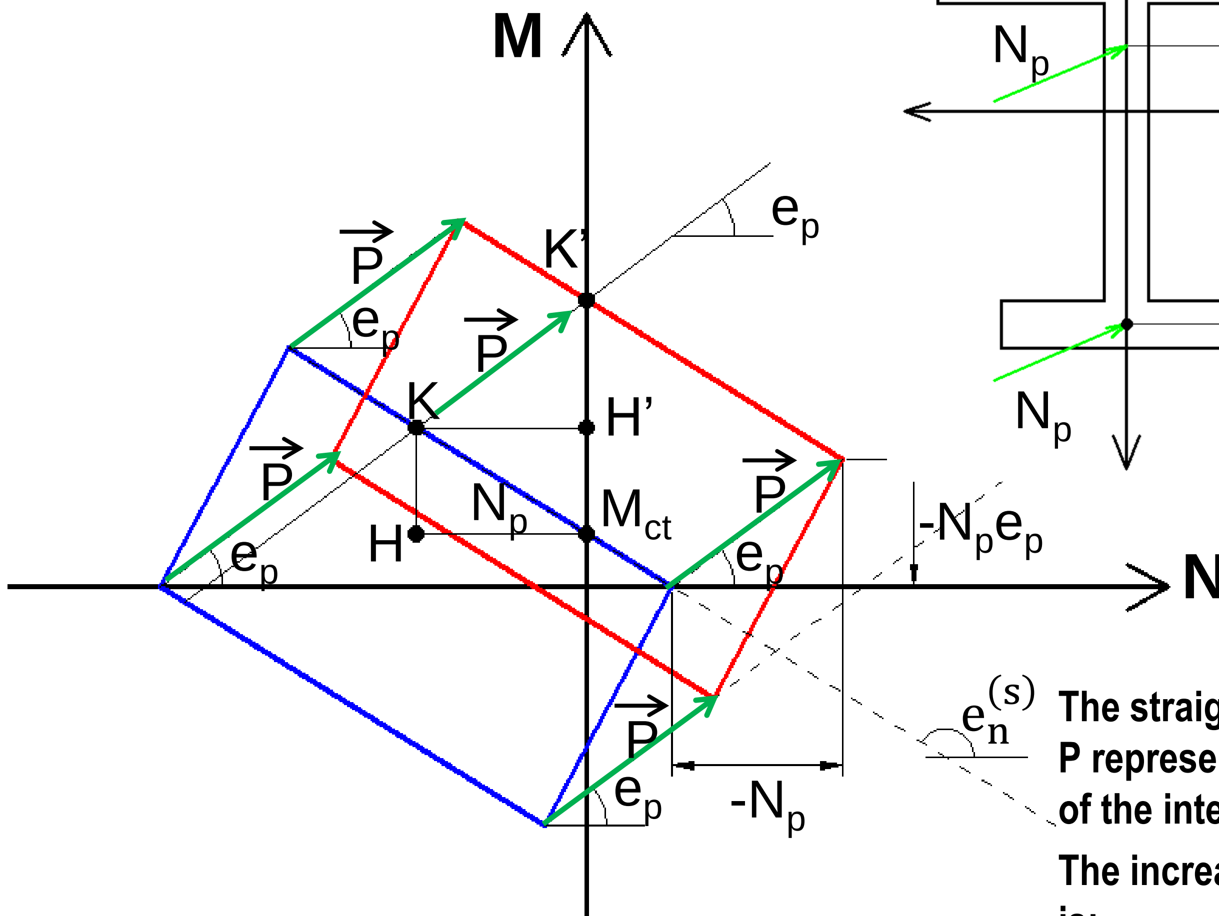
$$N \quad = N_0 - N_p$$

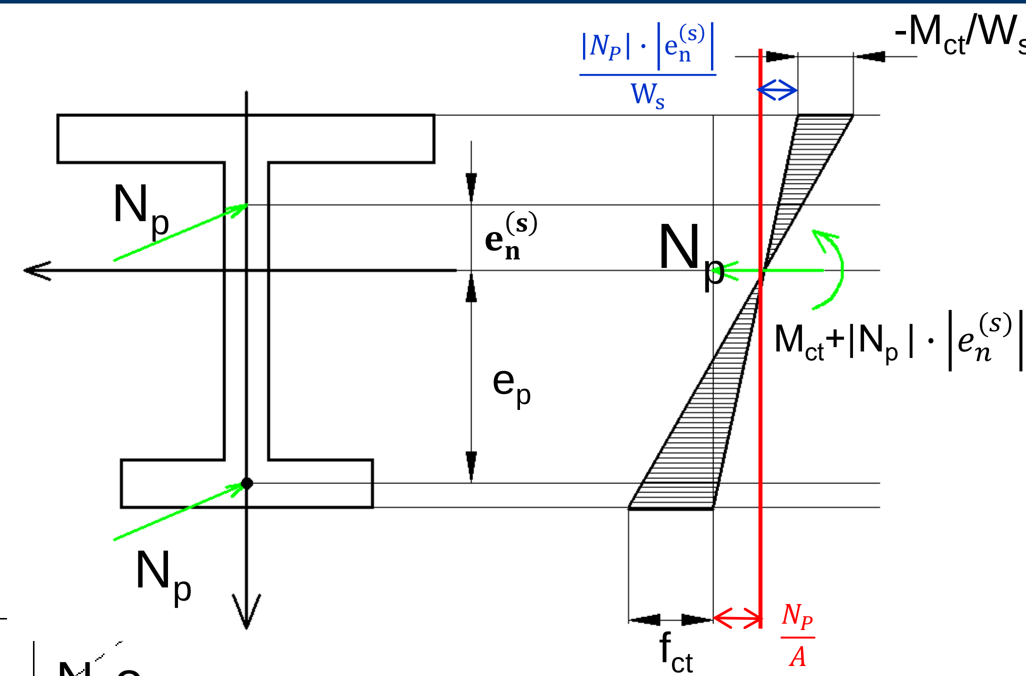
$$M \quad = M_0 - N_p e_p$$




The equilibrium must be satisfied with reference to a resisting domain translated by the vector $\vec{P}$ with components $-N_p$, $-N_p e_p$. Referring to the case of a compressive force $N_p$, with positive eccentricity $e_p$, the two components of the vector $\vec{P}$ are positive.

# THE MODIFIED INTERACTION DIAGRAM

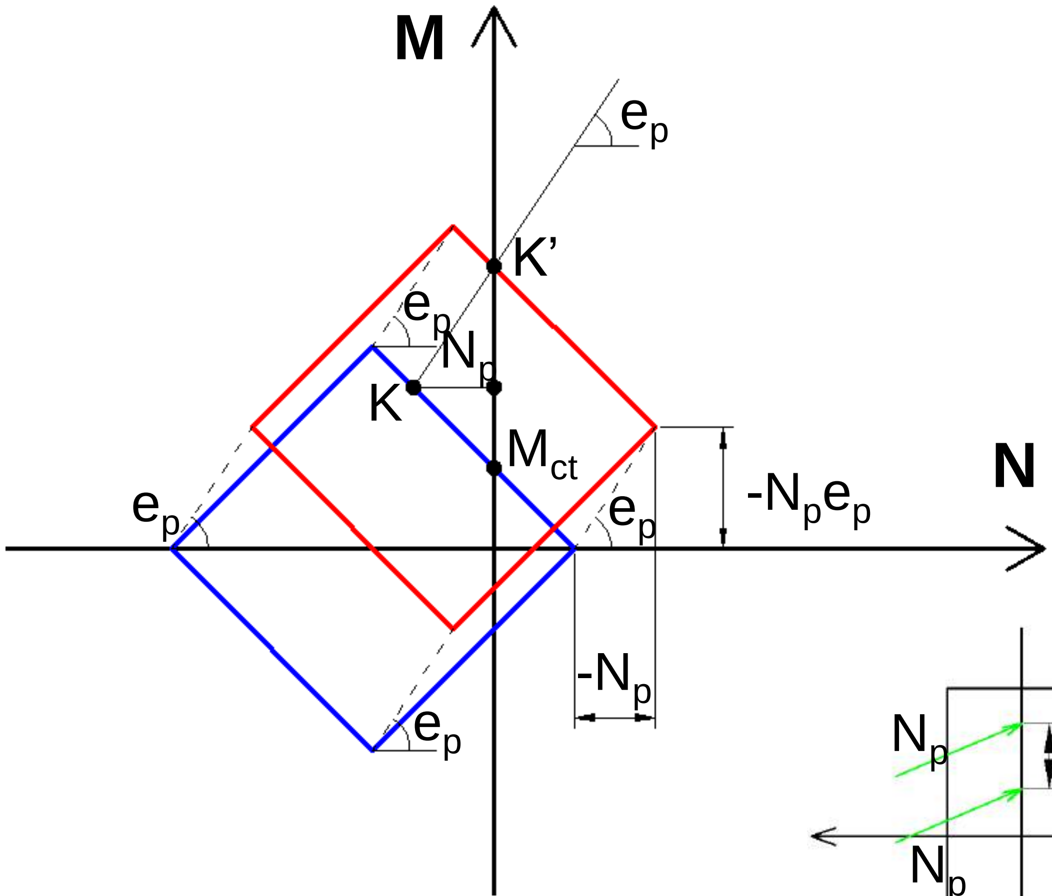



**The straight lines defining the direction of vector P represent the geometrical locus of the vertexes of the interaction diagram $M_0$-$N_0$ with varying $N_p$.**

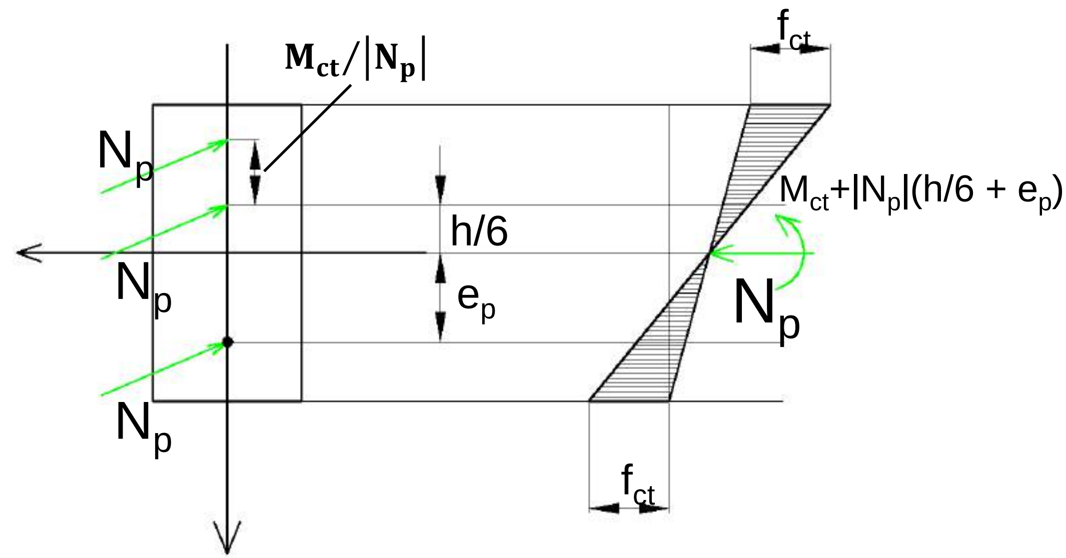
**The increase of the resisting bending moment $N_P$ is:**

$$\Delta M = M_{k'} - M_{ct} = |N_p| \left( \left| e_n^{(s)} \right| + e_p \right)$$




**Assuming $e_p = \beta h$ and remembering that $\left|e_n^{(s)}\right| = \frac{h}{6}$, the increase of the bending capacity due to $N_P$ is:**

$$\Delta M = M_{k'} - \mathbf{M_{ct}} = \left|\mathbf{N_p}\right|\mathbf{h}\left(\frac{\mathbf{1+6\beta}}{\mathbf{6}}\right)$$



# HOW TO APPLY PRESTRESSING

Prestressing, which consists in applying the forces $N_p, N_p e_p$, can be introduced in different ways, whose convenience depends on different factors:

- Reliability;
- Execution;
- Durability of the system;
- Durability of the induced stress pattern.

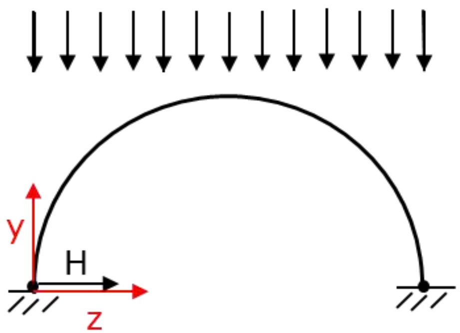
In the construction field, the application of an imposed relative elastic deformation between the structure and its restraints has traditionally been used.

In the special case of concrete sections, the imposed elastic deformation acts between the section and the reinforcement, which behaves as an elastic internal restraint. The external restraints can be statically determined or undetermined.

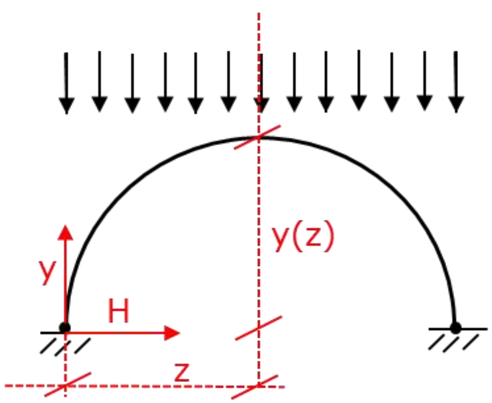
From the technology point of view, prestressing poses a few challenges, among which the most difficult one is to introduce a stress pattern that remains unaltered over time.

# THE INTRODUCTION OF PRESTRESSING IN STRUCTURAL SYSTEMS

Two hinges arch

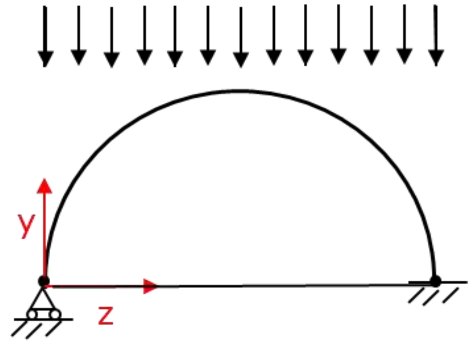


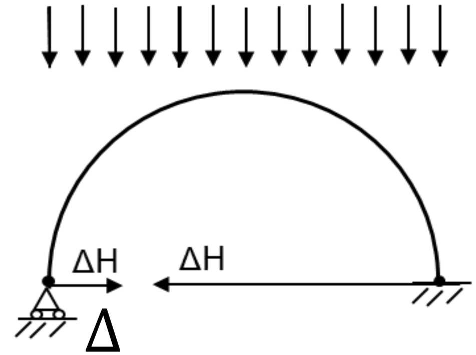

$$\mathbf{H} = -\frac{\boldsymbol{\delta}_0}{\boldsymbol{\delta}_c} = \mathbf{H}_R$$

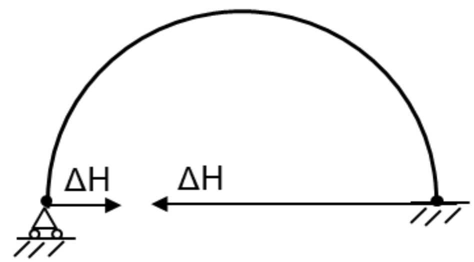



$$\mathbf{M(z)} = \mathbf{M_g(z)} - \mathbf{H_R} \cdot \mathbf{y(z)}$$

Tied arch




Load $\mathbf{H} = -\dfrac{\boldsymbol{\delta}_0}{\boldsymbol{\delta}_c + \boldsymbol{\delta}_s} = \mathbf{H}_R\boldsymbol{\omega}$

$$\boldsymbol{\omega} = \frac{\boldsymbol{\delta}_c}{\boldsymbol{\delta}_c + \boldsymbol{\delta}_s}$$

Unloaded arch H =0

Load + imposed deformation

$$\Delta\mathbf{H} = \frac{\Delta}{\boldsymbol{\delta}_c + \boldsymbol{\delta}_s}$$

$$\mathbf{H}_{tot} = \mathbf{H}_R\boldsymbol{\omega} + \frac{\Delta}{\boldsymbol{\delta}_c + \boldsymbol{\delta}_s} \geq \mathbf{H}_R$$

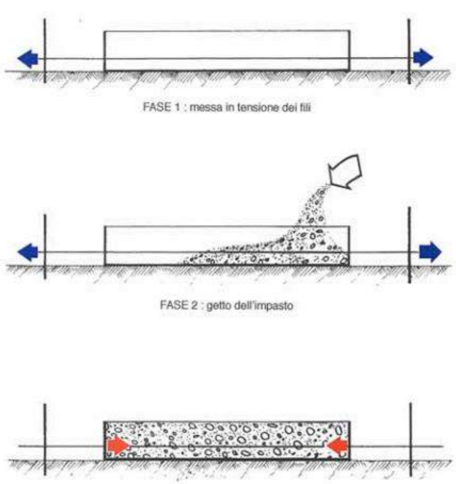
$$\Delta \geq \mathbf{H}_R(\mathbf{1} - \boldsymbol{\omega})(\boldsymbol{\delta}_c + \boldsymbol{\delta}_s) = \mathbf{H}_R\boldsymbol{\delta}_c\frac{(\mathbf{1} - \boldsymbol{\omega})}{\boldsymbol{\omega}}$$

Unloaded arch : $\mathbf{H}_{tot} = \dfrac{\Delta}{\boldsymbol{\delta}_c + \boldsymbol{\delta}_s}$

# THE INTRODUCTION OF PRESTRESSING IN R.C. BEAMS

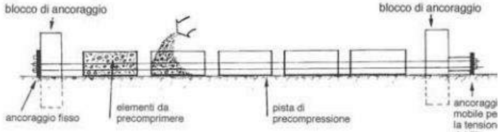
Specifically regarding reinforced concrete elements, prestressing is applied using the following techninques:

- Pre-tensioning (using bonded tendons);
- Post-tensioning, using tendons that are initially unboned and later bonded by means of injection (injected tendons post-tensioning);
- Post-tensioning by means of unbonded tendons. The tendons can be placed inside or outside the r.c. elements, in sheaths protected by a layer of grease.




FASI DELLA PRE-TENSIONE

Fig. 5




FASI DI PRE-TENSIONE

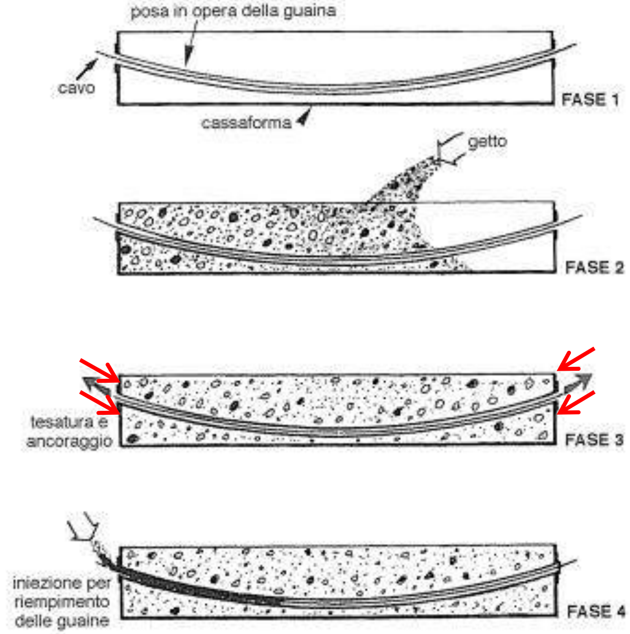
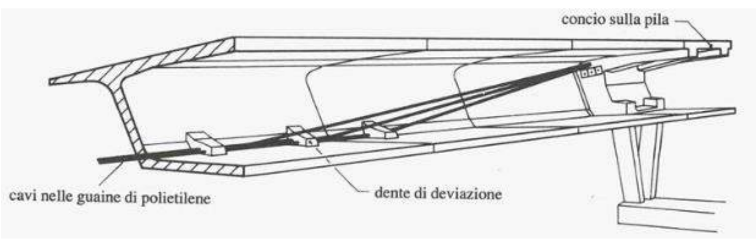
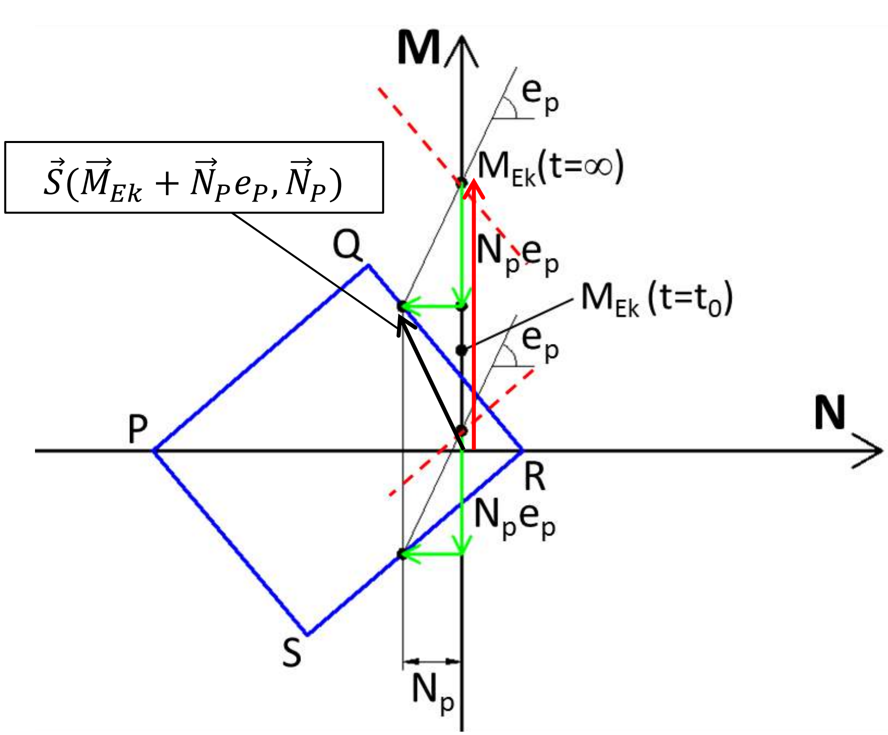


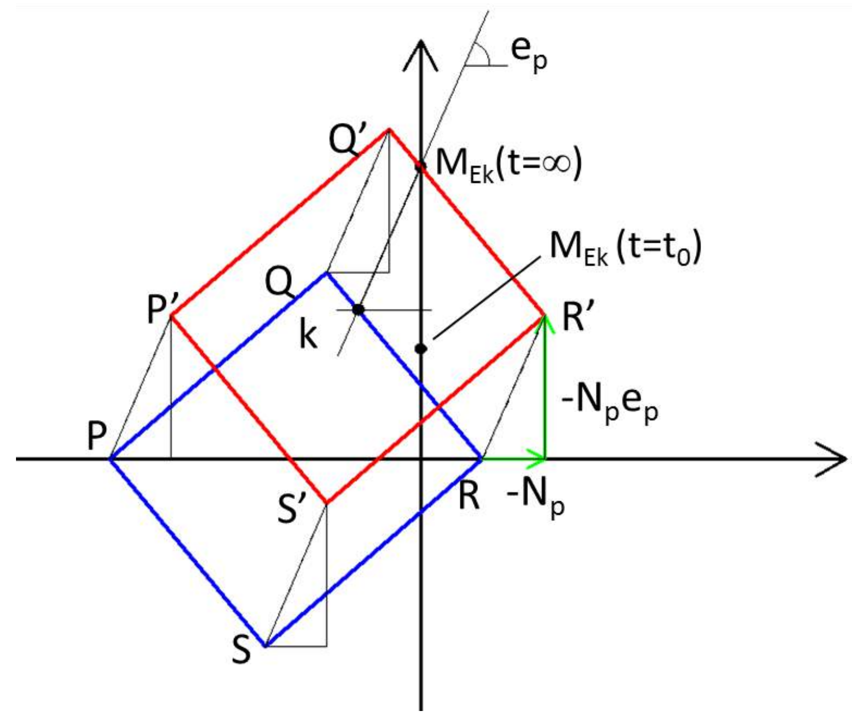

Fig. 10

FASI DELLA POST-TENSIONE




# THE TWO ALTERNATIVE APPROACHES TO POST-TENSIONING




**A) EXTERNAL FORCE**

**This approach is useful with particular reference to the analysis of unbonded cables at ultimate limit state.**

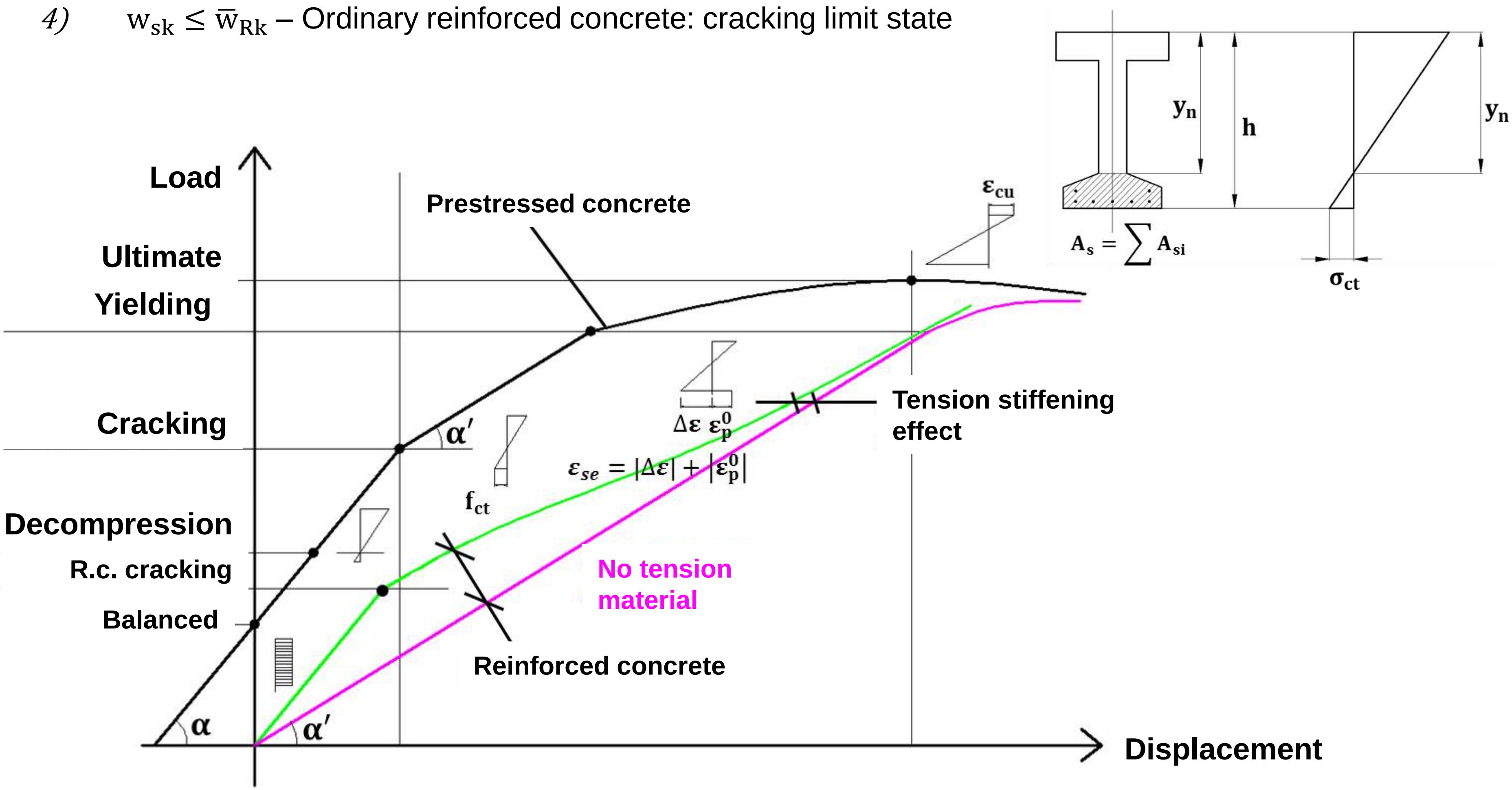
**B) TRANSFORMED M-N INTERACTION DIAGRAM**

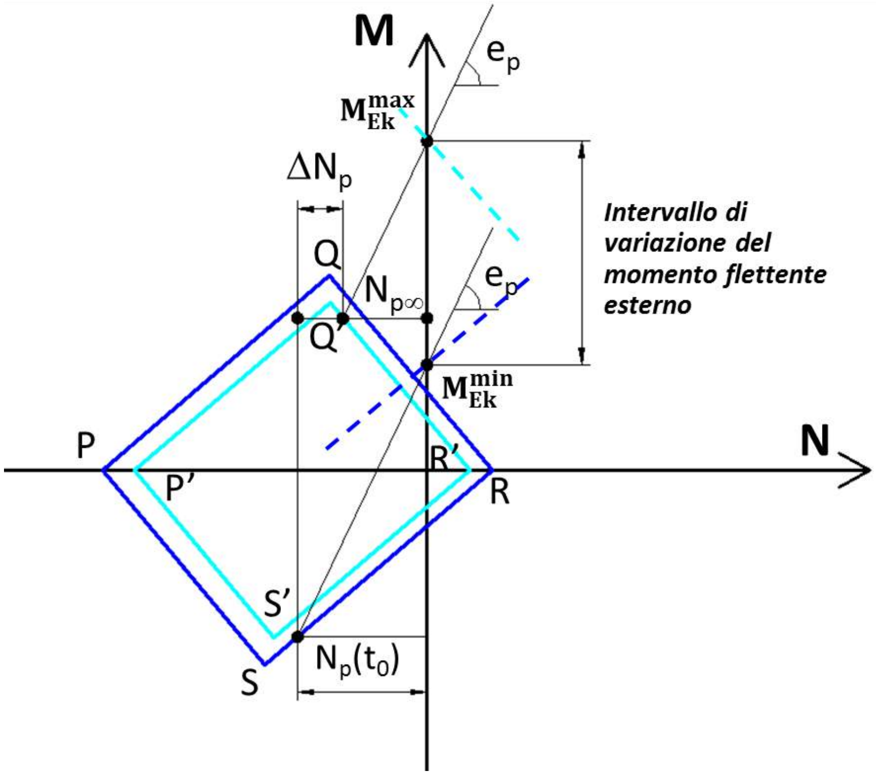
**The basic parallelogram (PQRS) is transforemed in the (P'Q'R'S') one by applying the vector with components ($-N_p$, $-N_p e_p$).**

**This method can be used for sectional analysis of pretrensioned elemets at ultimate.**

# THE 4 VERIFICATION CLASSES

1) $\sigma_C \leq 0$ – Integral prestressing
2) $\sigma_{ct} \leq (f_{ct}, f'_{ct})$ – Prestressing with limited tension, $\int_{y_n}^{h} \sigma_c(y) b(y) dy = \bar{\sigma}_s A_s$
3) $M \geq M_{cr}(f_{ct}, N_p)$, Rare combination – Partial prestressing, control of cracking
4) $w_{sk} \leq \bar{w}_{Rk}$ – Ordinary reinforced concrete: cracking limit state






$$\mathbf{Dominio\ (P, Q, R, S), f_c, f_{ct}, t = t_0}$$

$$\mathbf{Dominio\ (P', Q', R', S'), f'_c, f'_{ct}, t = \infty}$$

- **Some Code prescriptions:**

$t = t_0$ | $t = \infty$

$|f_c| = 0.6 \cdot f_{ck}$; $|f'_c| = 0.45 \cdot f_{ck}$, (DM 2018)

$f_{ct} = 0.1 \cdot f_{ck}$; $f'_{ct} = 0.07 \cdot f_{ck}$ (Good practice)

$f_{ct} = 0$ – No tension (integral prestressing)

**ACI Code, t = $t_0$:**

$|f_c| = 0.6 \cdot f_{ck}$ (Pre – tensioning) $|f_c| = 0.55 \cdot f_{ck}$ (Post – tensioning)

$|f_{ct}| = 1.4$ MPa; $|f_{ct}| = 0.25 \cdot \sqrt{f_{ck}}$ (unbonded cables)

$f_{ct,max} = 0.625 \cdot \sqrt{f_{ck}}$ (grouted cables)

**ACI Code, t = ∞:**

$|f_c| = 0.4 \cdot f_{ck}$

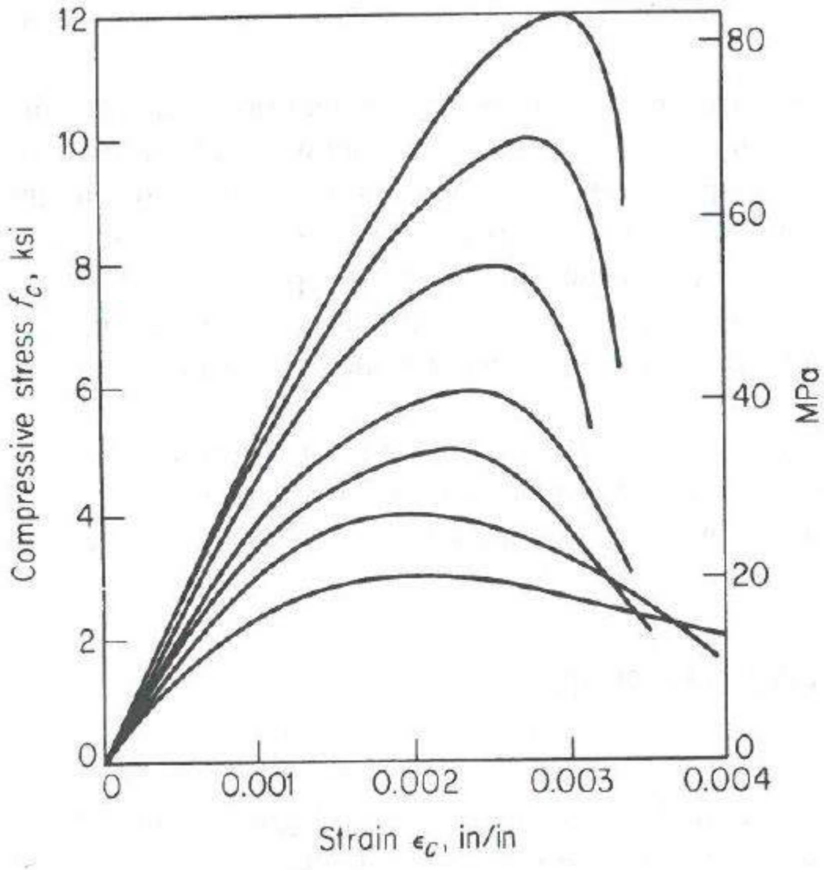
$|f_{ct}| \leq 0.5 \cdot \sqrt{f_{ck}}$ (grouted cables)

$|f_{ct}| \leq 0.25 \cdot \sqrt{f_{ck}}$ (severe exposure classes)

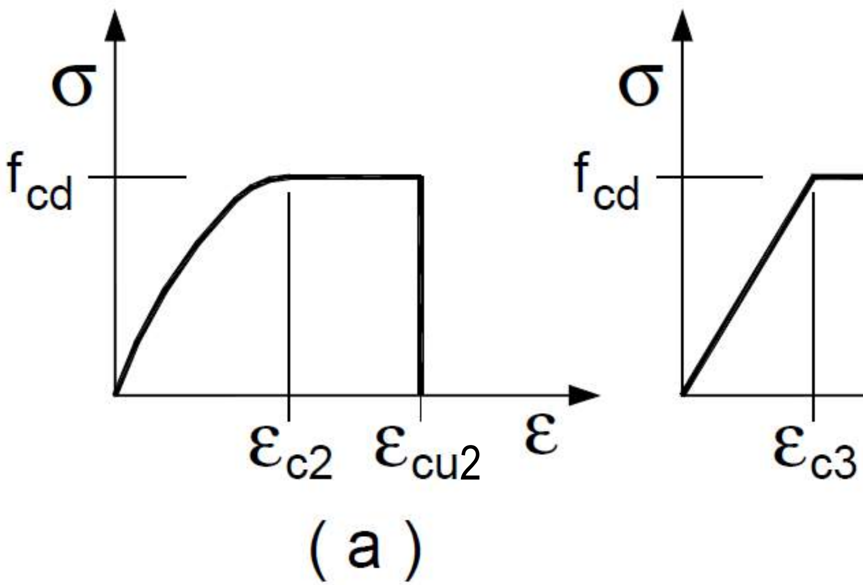
$f_{ct} = 0$ (unbonded tendons)

# CONCRETE: CONSTITUTIVE LAWS

**Behaviour of concrete under uniaxial stress:**



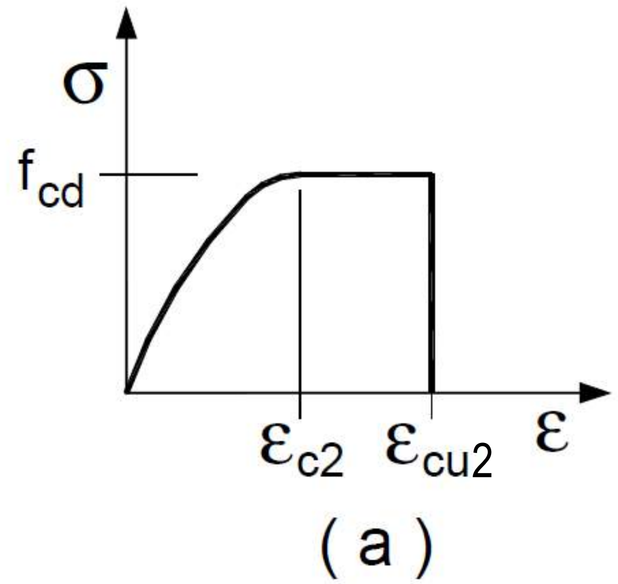
- The constitutive laws used in the the design of cross sections for bending and axial load can be of three different types:
  a) Exponential function
  b) Bilinear function
  c) Rectangular stress distribution



## (a) Exponential function:

$$\sigma_c = f_{cd} \cdot \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right];\ 0 \le \varepsilon_c \le \varepsilon_{c2}$$

$$\sigma_c = f_{cd}\ ;\quad \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2}$$




Where:

$$\varepsilon_{c2} = 2‰\ ,\ f_{ck} \le 50\ MPa$$

$$\varepsilon_{c2} = 2‰ + 0.085 \cdot \frac{(f_{ck} - 50)^{0.53}}{1000}\ ;\quad f_{ck} > 50\ MPa$$

$\varepsilon_{cu2} = 3.5‰$ n = 2 $f_{ck} \le 50\ MPa$

$$\varepsilon_{cu2} = 2.6‰ + 35/1000 \cdot \left[\frac{(90 - f_{ck})}{100}\right]^4$$

$$f_{ck} > 50\ MPa$$

$$n = 1.4 + 23.4 \cdot \left[\frac{(90 - f_{ck})}{100}\right]^4$$

In the new IBC n = 2

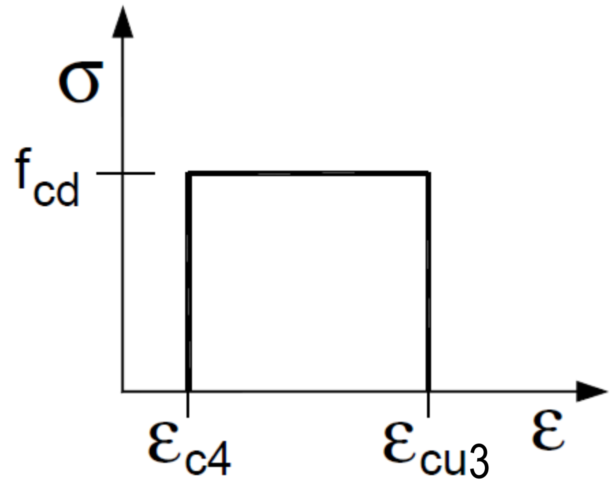
## (c) Rectangular stress distribution:

$$\lambda = 0.8\ ,\quad f_{ck} \leq 50\ MPa$$

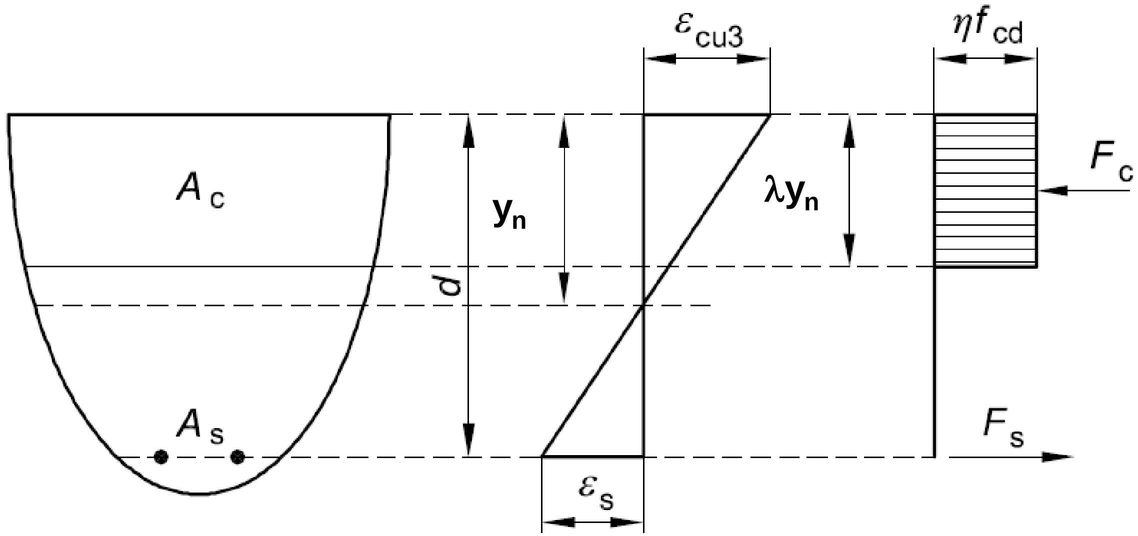
$$\lambda = 0.8 - \frac{(f_{ck}-50)}{400}\ ,\quad 50MPa \leq f_{ck} \leq 90\ MPa$$

$$\eta = 1\ ,\quad f_{ck} \leq 50\ MPa$$

$$\eta = 1 - \frac{(f_{ck}-50)}{200}\ ,\quad 50MPa \leq f_{ck} \leq 90\ MPa$$

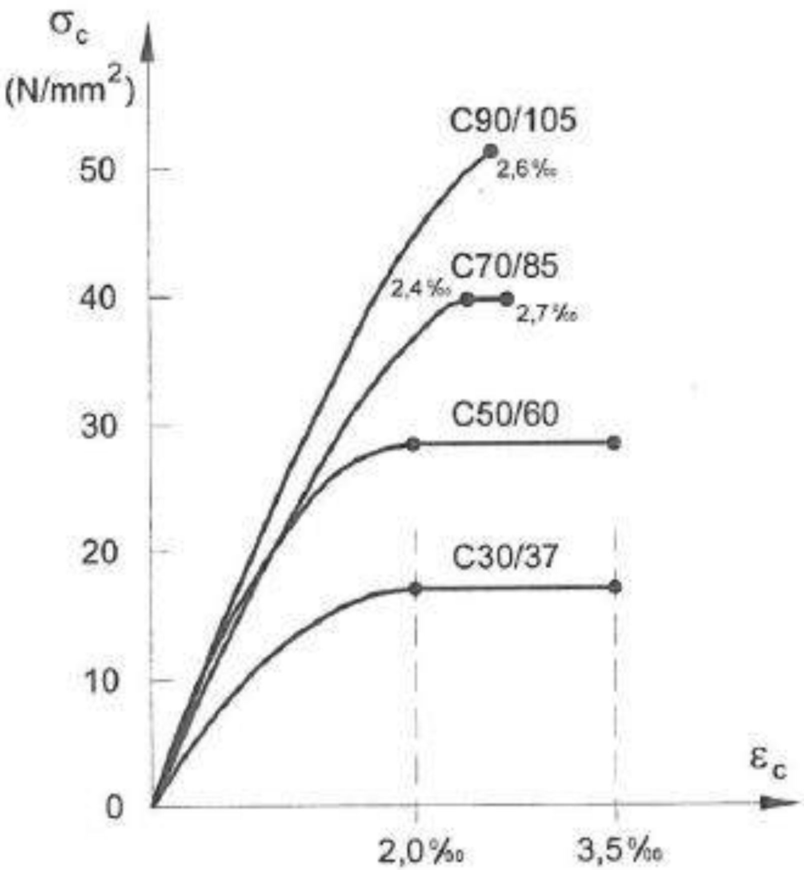



( c )

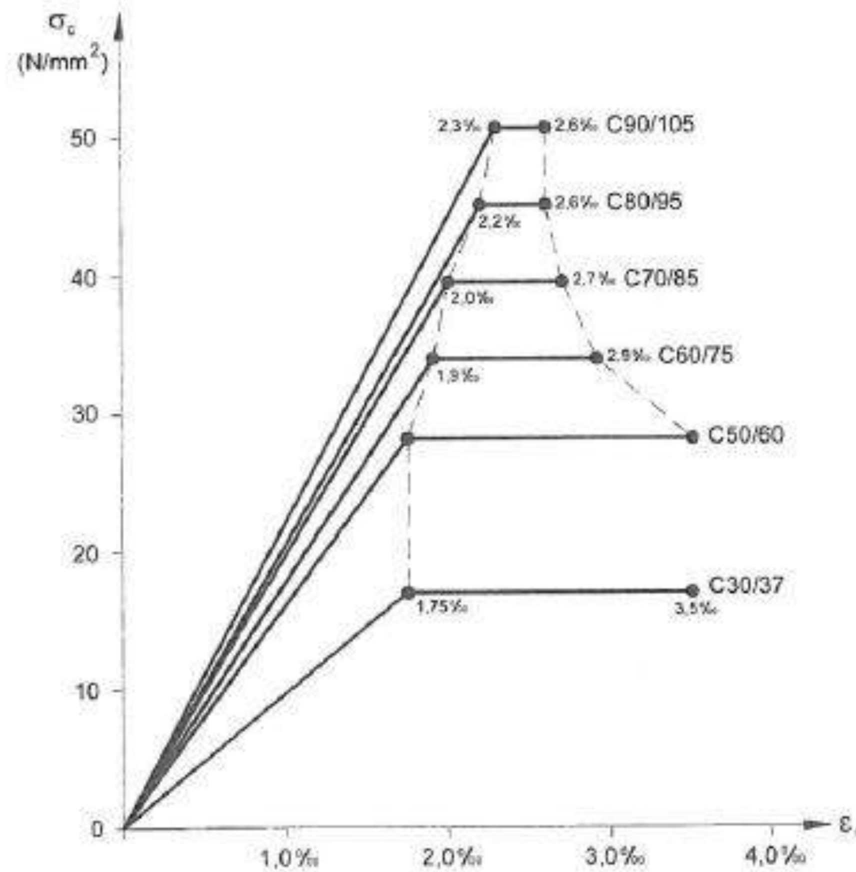



$$\varepsilon_{c4} = (1-\lambda)\cdot\varepsilon_{cu3}$$

# CONCRETE: IDEALIZED CONSTITUTIVE LAWS



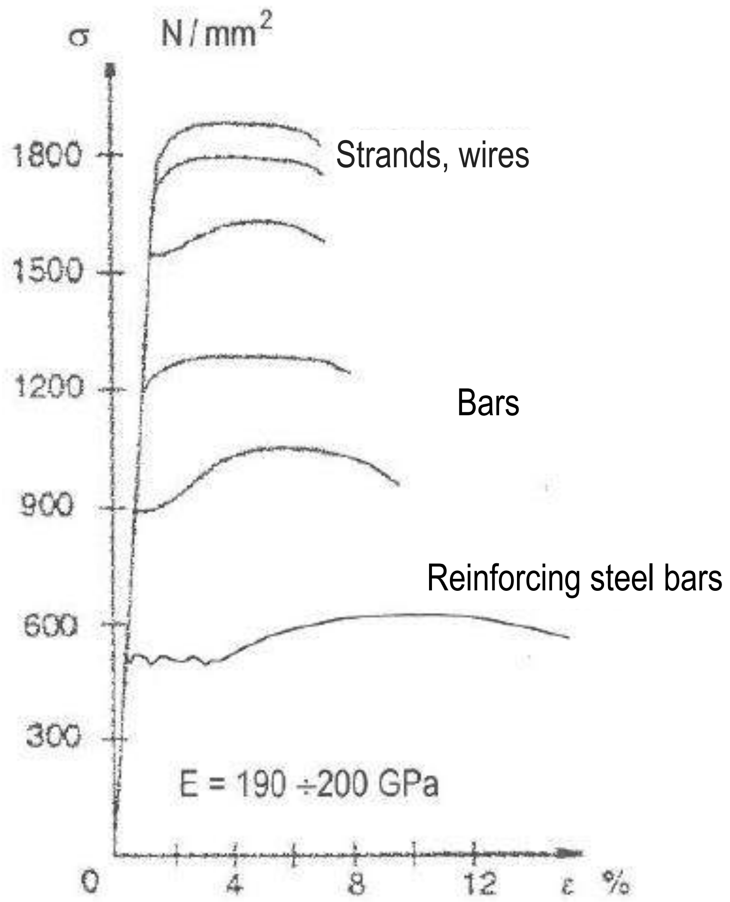

(a)

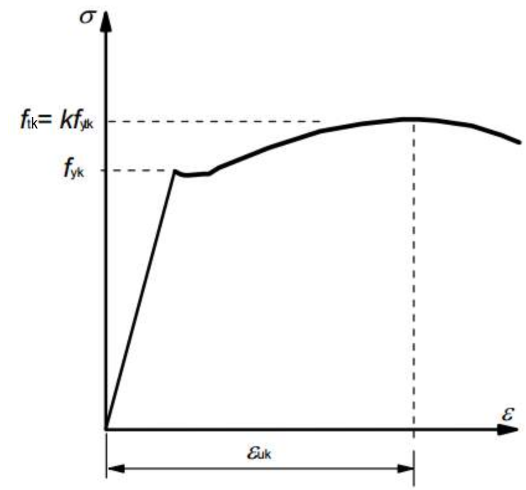



(b)

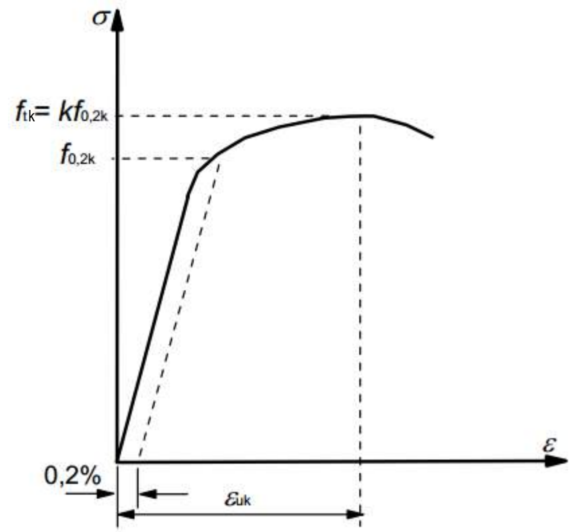
# STEEL CONSTITUTIVE LAW

## Stress-strain diagrams



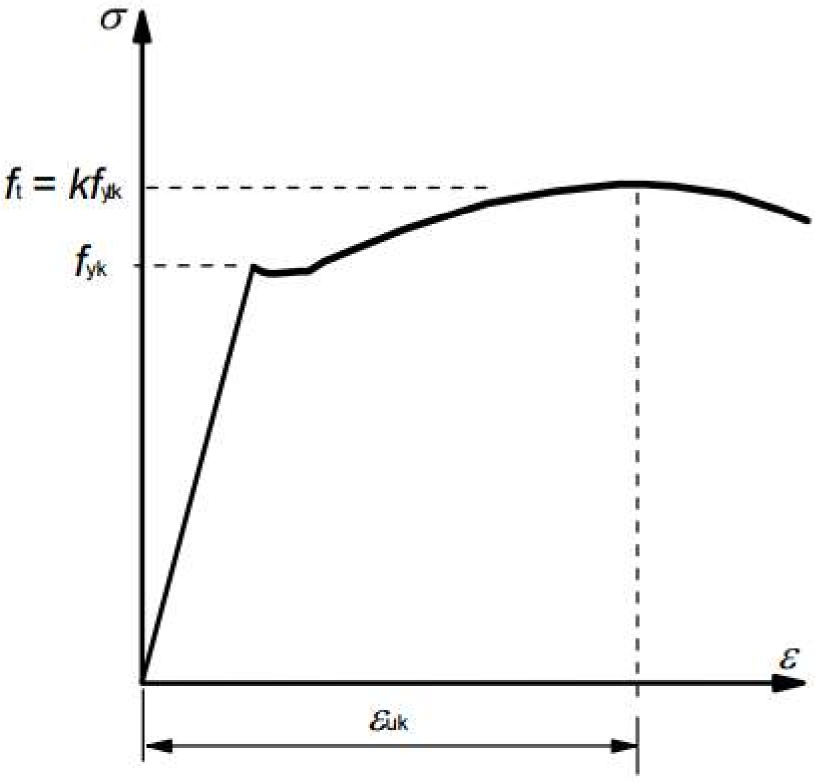




**a) Hot rolled steel**

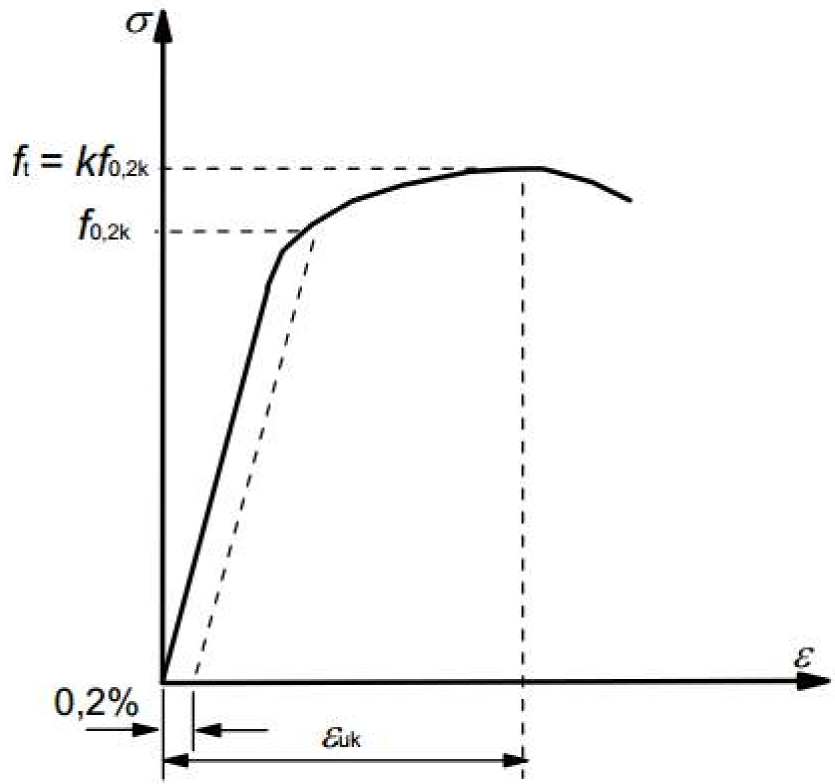



**b) Cold worked steel**

# REINFORCING STEEL: Stress-strain diagrams




**a) Hot rolled steel**




**b) Cold worked steel**

# PRESTRESSING STEEL

- This clause applies to wires, bars and strands used as prestressing tendons in concrete structures.

- The prestressing tendons (wires, strands and bars) shall be classified according to:
  (i) Strength, denoting the value of the 0.1% proof stress ($fp_{0,1k}$) and the value of the ratio of tensile strength to proof strength ($f_{pk}$ /$fp_{0.1k}$) and elongation at maximum load ($\varepsilon_{uk}$) .
  (ii) Class, indicating the relaxation behaviour .
  (iii) Size.
  (iv) Surface characteristics.

- The design value for the modulus of elasticity, $E_p$ may be assumed equal to 205 GPa for wires and bars. The actual value can range from 195 to 210 GPa, depending on the manufacturing process.

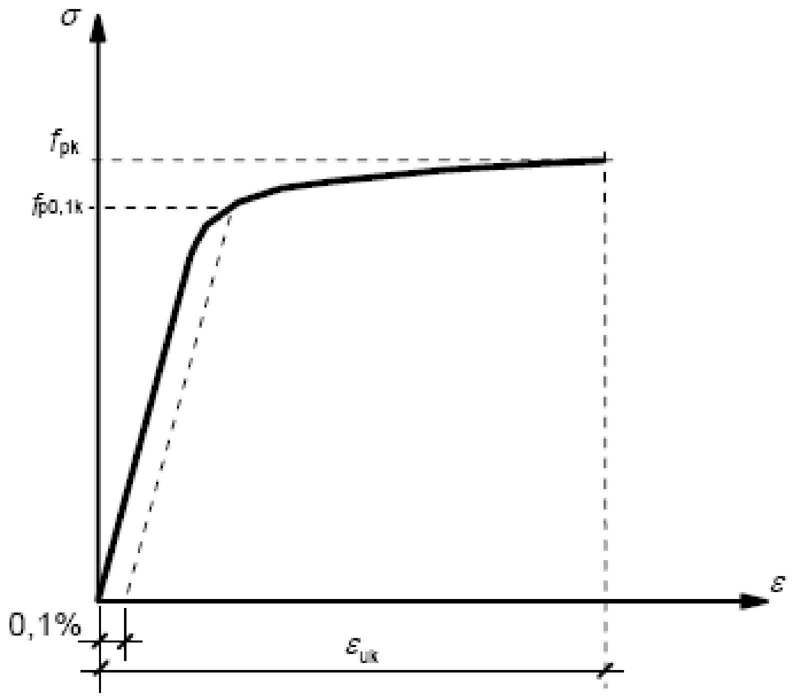
- The design value for the modulus of elasticity, Ep may be assumed equal to 195 GPa for strand. The actual value can range from 185 GPa to 205 GPa.

- The mean density of prestressing tendons for the purposes of design may normally be taken as 7850 kg/m$^3$

- The values given above may be assumed to be valid within a temperature range between -40°C and +100°C for the prestressing steel in the finished structure.

# PRESTRESSING STEEL

The 0.1% proof stress ($f_{p0.1k}$) and the specified value of the tensile strength ($f_{pk}$) are defined as the characteristic value of the 0.1% proof load and the characteristic maximum load in axial tension respectively, divided by the nominal cross sectional area, as shown in this figure:




**Stress-strain diagram for typical prestressing steel (absolute values are shown for tensile stress and strain)**

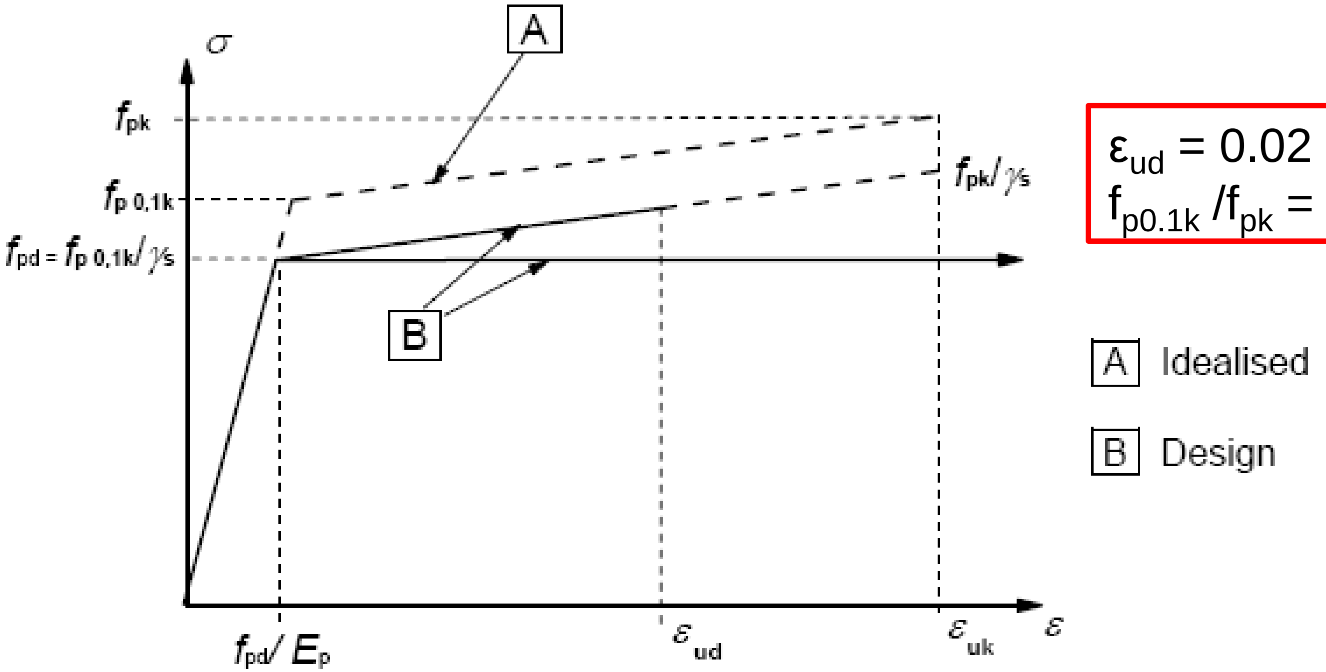
# PRESTRESSING STEEL

The design value for the steel stress can be taken as:

$f_{pd} = f_{p0,1k}/\gamma_S$

For cross-section design, either of the following assumptions may be made :

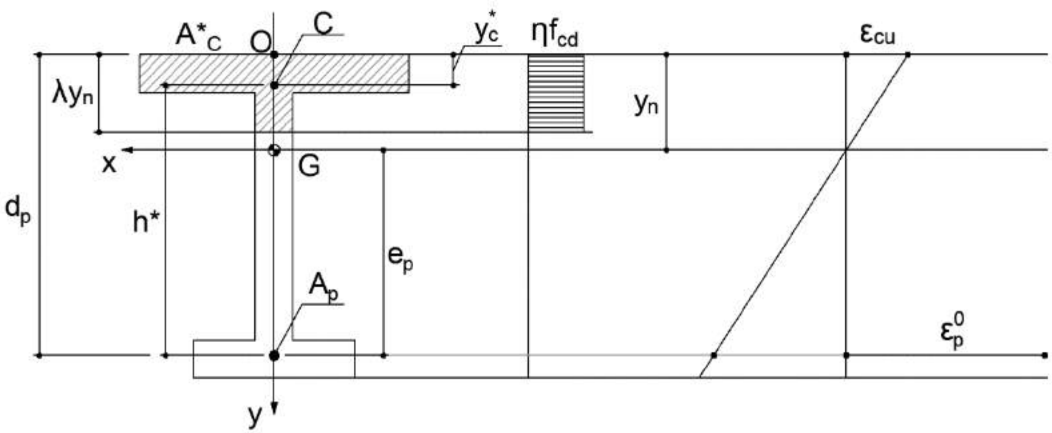
- an inclined branch, with a strain limit $\varepsilon_{ud}$. Or
- a horizontal top branch without strain limit.




$\varepsilon_{ud} = 0.02$
$f_{p0.1k} / f_{pk} = 0.9$

# THE ULTIMATE LIMIT STATE FOR BENDING AND AXIAL FORCE

- **Prestressed Concrete – Unbonded cables**




$$\sigma_p = \sigma_p^0 = E_p \cdot \varepsilon_p^0$$

Translational Equilibrium:

$$-\eta f_{cd} \cdot A_c^* = -\sigma_p^0 \cdot A_p \quad \rightarrow A_c^* = \frac{\sigma_p^0 A_P}{\eta f_{cd}}$$

Rotational Equilibrium:

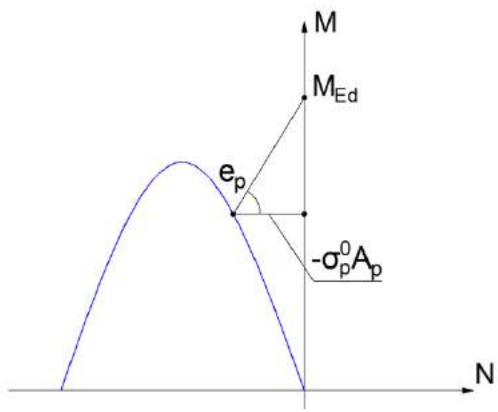
$$\sigma_p^0 \cdot A_p \cdot h^* = M_{Ed}$$

For rectangular sections:

$$\lambda b y_n = \frac{\sigma_p^0 A_p}{\eta f_{cd}}$$

$$\sigma_p^0 \cdot A_p \cdot \left(d_p - y_c^*\right) = M_{Ed}$$

$$y_c^* = \lambda y_n / 2$$



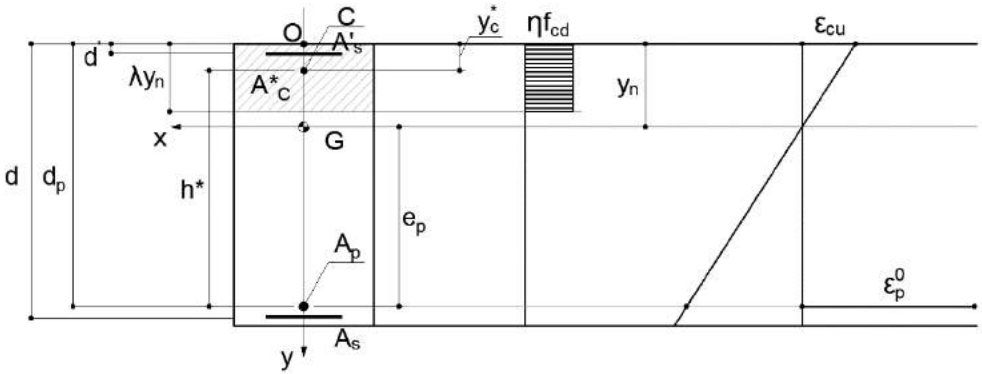

Introducing the non-dimensional quantities:

$$\xi = \frac{y_n}{h}; \quad \alpha_p = \frac{\sigma_p^0}{f_{yd}}; \quad \omega_p = \frac{f_{yd} A_p}{bh \cdot \eta f_{cd}}; \quad \mu_{Ed} = \frac{M_{Ed}}{bh^2 \cdot \eta f_{cd}}; \quad \delta_p = \frac{d_p}{h} \quad \longrightarrow$$

$$\boxed{\begin{aligned} &\lambda \xi = \alpha_p \omega_p \\ &\alpha_p \omega_p \cdot \left(\delta_p - \frac{\alpha_p \omega_p}{2}\right) = \mu_{Ed} \end{aligned}}$$

# ULTIMATE LIMIT STATE: SECTIONAL CAPACITY FOR UNBONDED TENDONS AND ORDINARY REINFORCEMENT

## A) UNBONDED CABLES – RECTANGULAR SECTIONS



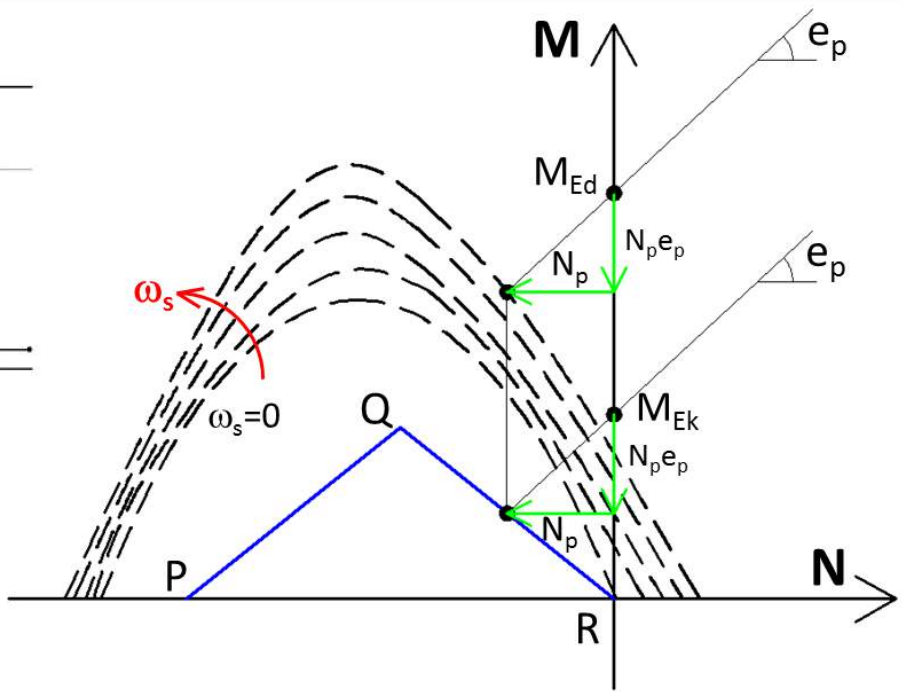

$$\lambda\xi = \alpha_p\omega_p + \omega_s(1-\beta)$$

$$\alpha_p\omega_p \cdot \left(\delta_p - \frac{\lambda\xi}{2}\right) + \omega_s \cdot \left[\left(\delta - \frac{\lambda\xi}{2}\right) - \beta\left(\delta' - \frac{\lambda\xi}{2}\right)\right] = \mu_{Ed}$$

$$\omega_s = \frac{A_s f_{yd}}{\eta f_{cd} bh}$$

$$A'_s = \beta A_s$$

## B) UNBONDED CABLES, THE EFFECT OF THE ORDINARY REINFORCEMENT




**To increase sectional capacity at ultimate, ordinary reinforcement ($\omega_s$) can be conveniently added. In this way, the safety level and the sectional ductility can be significantly increased.**

# ULTIMATE BENDING LIMIT STATE

- **Prestressed concrete – Bonded cables**

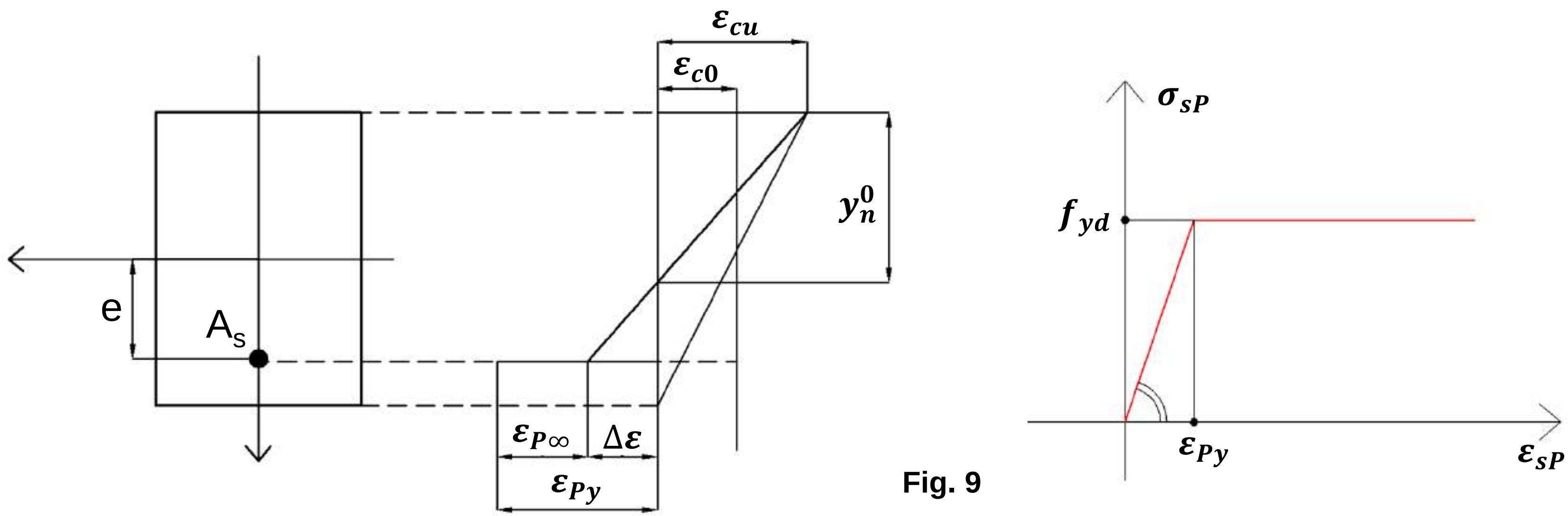



Fig. 9

When the cable is bonded, the steel tension varies when varying the distribution of strains at ultimate. We so distinguish various zones:

- $0 \le y_n \le y_n^0$ $\rightarrow$ $\Delta\varepsilon \ge \varepsilon_{Py} - \varepsilon_{P\infty}$ the steel is at yielding state
- $y_n^0 \le y_n \le h$ $\rightarrow$ $\Delta\varepsilon \le \varepsilon_{Py} - \varepsilon_{P\infty}$ the steel is in the elastic state
- $y_n \to \infty;$ $\varepsilon_{sP} = \varepsilon_{P\infty} + \varepsilon_{c0}$

To obtain the position of $y_n^0$ we write: $\frac{|\varepsilon_{cu}|}{y_n^0} = \frac{\Delta\varepsilon}{d - y_n^0}$ *so that* $\frac{y_n^0}{h} = \xi^0 = \delta \cdot \frac{1}{1 + \frac{\Delta\varepsilon}{|\varepsilon_{cu}|}}$

# ULTIMATE BENDING LIMIT STATE

- **Prestressed concrete – Bonded cables**

$$\Delta\varepsilon = \varepsilon_{py} - \varepsilon_p^0 = \varepsilon_{py}(1-\alpha_p), \qquad \alpha_p = \frac{\varepsilon_p^0}{\varepsilon_{py}}$$

$$\xi^0 = \frac{\delta}{1 + k_p(1-\alpha_p)}, \qquad k_p = \frac{\varepsilon_{py}}{|\varepsilon_{cu}|}$$

(1)
$$\begin{cases} \lambda\xi = \omega_p \\ \omega_p\left(\delta_p - \dfrac{\omega_p}{2}\right) = \mu_{Ed} \end{cases} ;\ \omega_p = \frac{f_{yp}A_p}{bh\cdot\eta f_{cd}};\quad \omega_{p,lim} = \lambda\xi^0 = \frac{\lambda\delta}{1+k_p(1-\alpha_p)};\quad \mu_{Ed} = \frac{M_{Ed}}{bh^2\cdot\eta f_{cd}}$$

If $\omega_p \le \omega_{p,lim},\ \xi \le \xi^0$ the reinforcement is in the yielded state, eqs. (1) have to be used

If $\omega_p > \omega_{p,lim}$

$$\sigma_p = E_p\cdot(\varepsilon_p^0 + \Delta\varepsilon) = E_p\cdot\left[\varepsilon_p^0 + |\varepsilon_{cu}|\frac{(d_p - y_n)}{y_n}\right]$$

$$\beta = \frac{\sigma_p}{f_{yp}} = \alpha_p + \frac{1}{k_p}\frac{(\delta_p - \xi)}{\xi} = \frac{(k_p\alpha_p - 1)\xi + \delta_p}{k_p\xi}$$

(2)
$$\begin{cases} -\lambda\xi + \beta\omega_p = 0 \\ \beta\omega_p\left(\delta_p - \dfrac{\beta\omega_p}{2}\right) = \mu_{Ed} \end{cases}$$

Introducing β in the first of eqs. (2) we reach:

$$\xi^2 - \frac{\omega_p}{\lambda k_p}[k_p\alpha_p - 1]\xi - \frac{\omega_p\delta_p}{\lambda k_p} = 0$$

# ULTIMATE BENDING LIMIT STATE

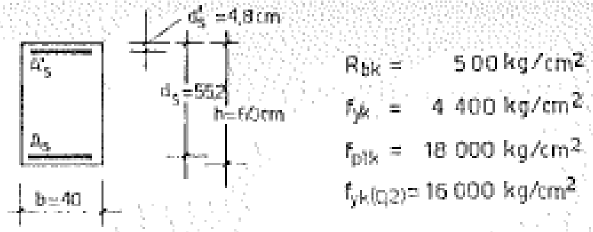



FIG. 1 - Caratteristiche geometriche e meccaniche della sezione trasversale.

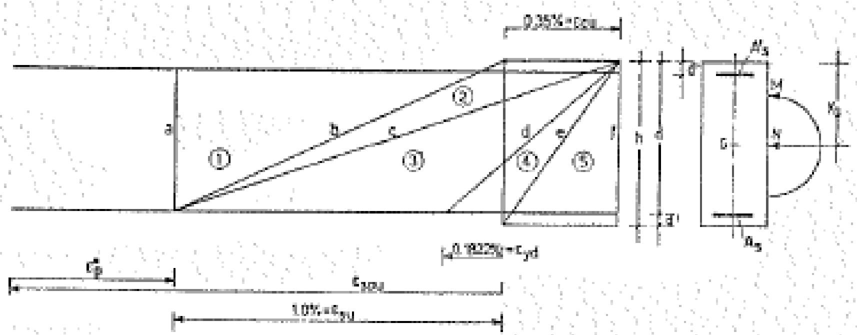



FIG. 2 - Campi di rottura.

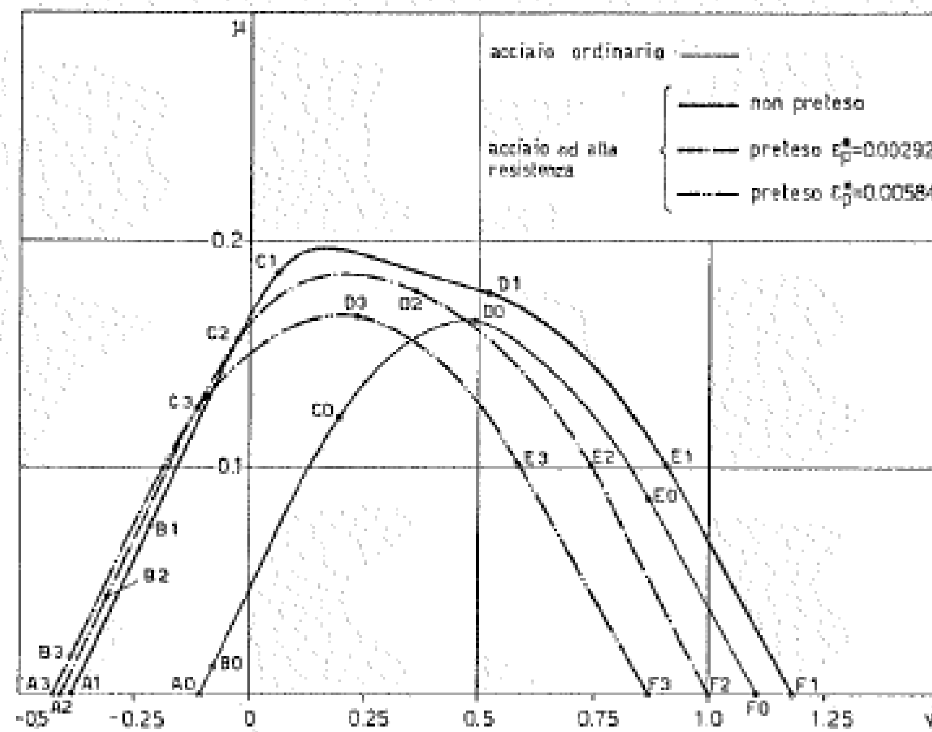



FIG. 3 - Diagrammi di interazione per la sezione di Fig.1 ( $A'_s = A_s = 7.35\ cm^2$ ).

# ULTIMATE BENDING LIMIT STATE

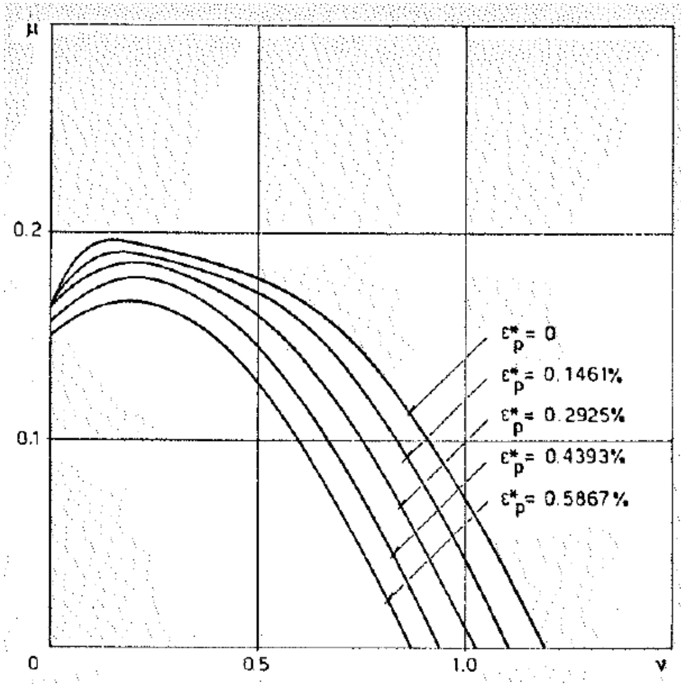



FIG. 12 - Diagrammi di interazione per la sezione di Fig. 1 al variare del livello di precompressione ($A_s = A'_s = 7.35\ cm^2$).

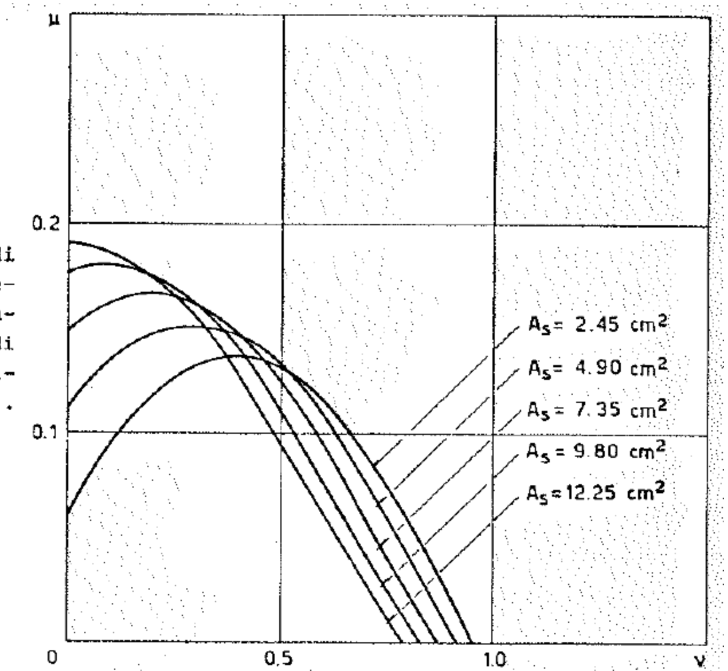
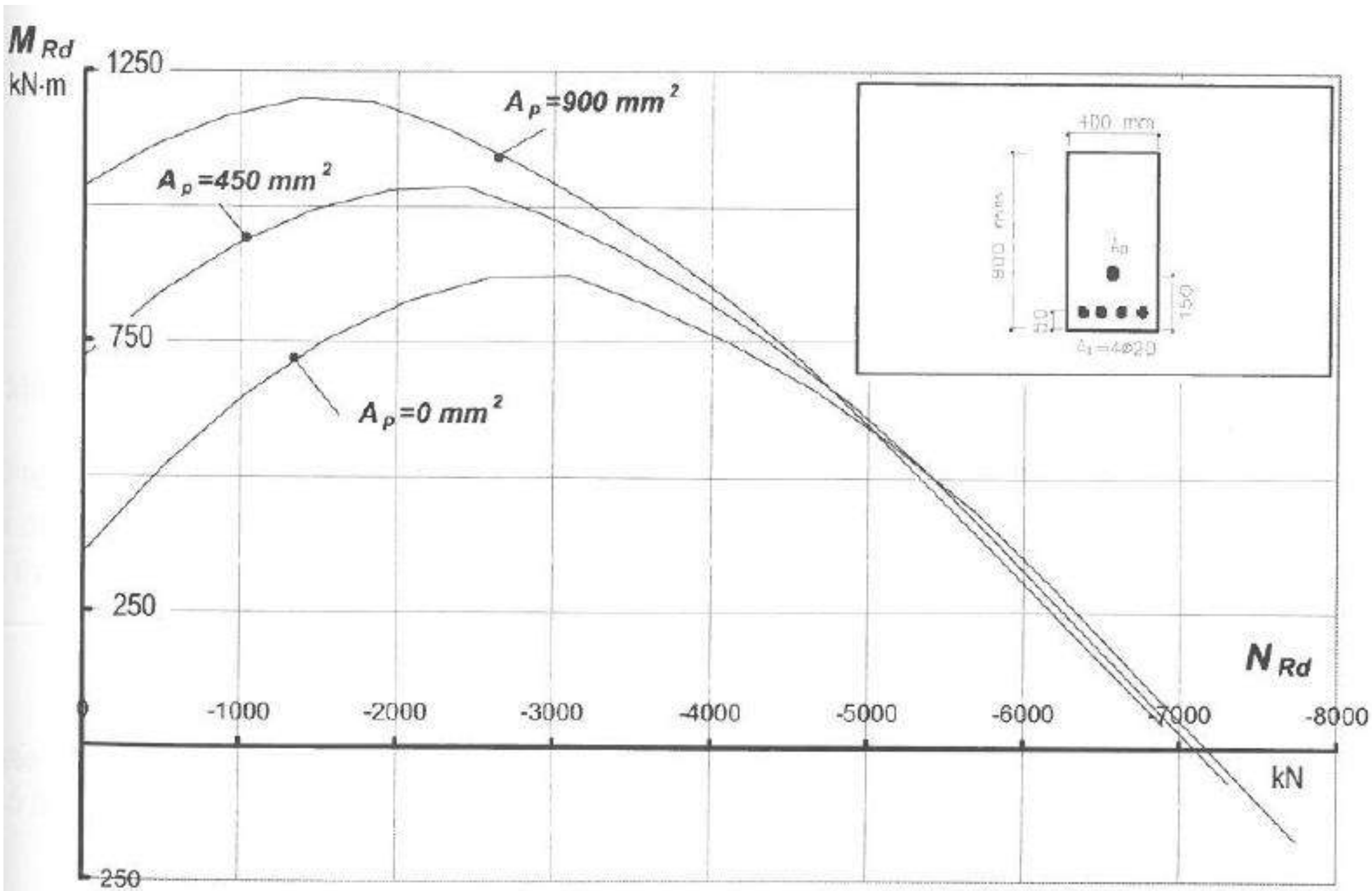


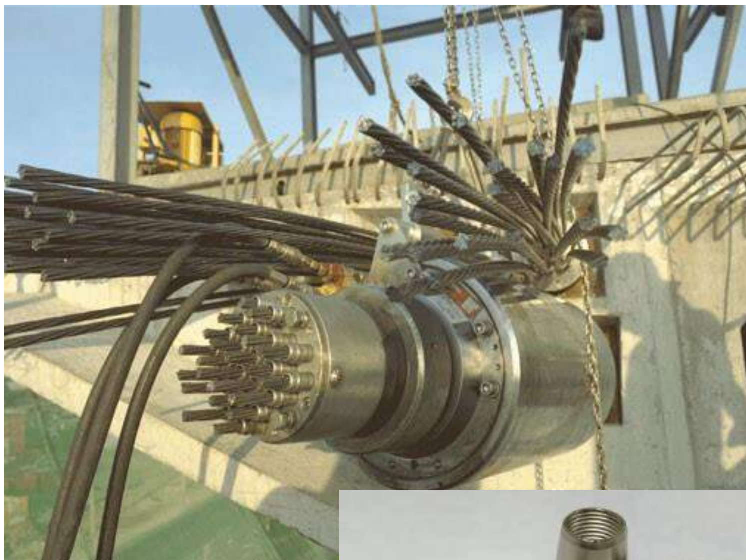

FIG. 13 - Diagrammi di interazione per la sezione di Fig. 1 al variare della quantità di acciaio ad alta resistenza ($\varepsilon^*_p = 0.00584$).

# ULTIMATE BENDING LIMIT STATE



# BASIC ASPECTS OF PRESTRESSING TECHNOLOGY

Steel strand

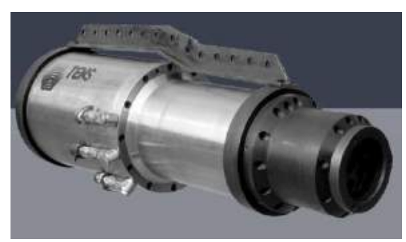


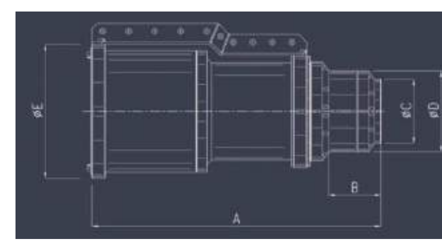
Wedges

Ducts

Bars

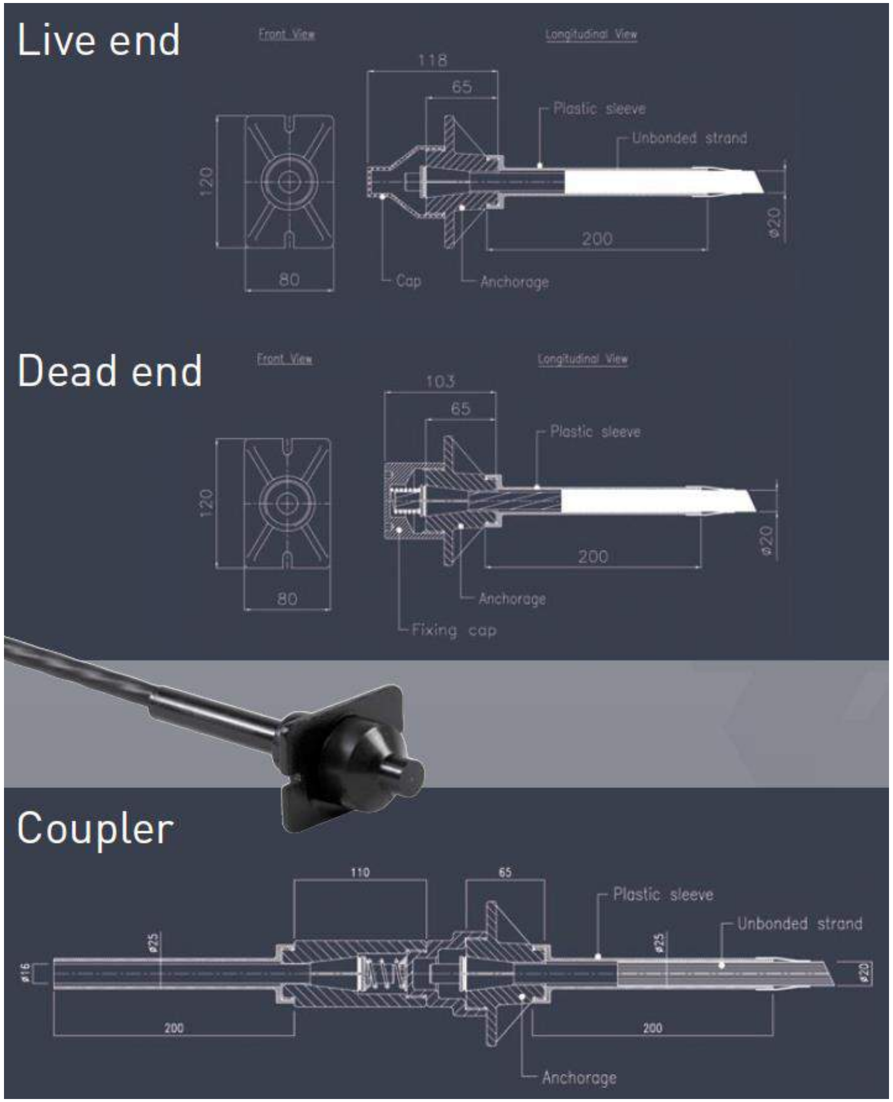

Live end and dead end





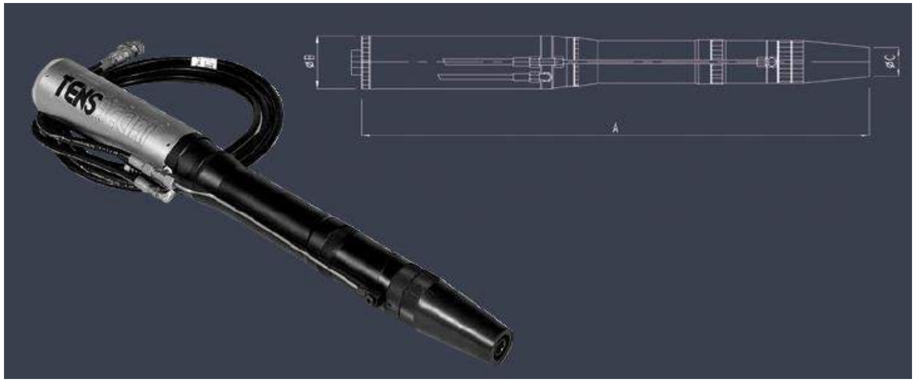
Multi strand stressing jacks

# BASIC ASPECTS OF PRESTRESSING TECHNOLOGY




Live end, dead end and coupler

Plant floor



Monostrand stressing jacks

# BASIC ASPECTS OF PRESTRESSING TECHNOLOGY

## Calculation notes

### Friction losses

The calculation of loss of prestressing force due to friction and wobble effects inside tendons is usually made starting from the following equation, taken from EN 1992-1-1:

$$\Delta P_\mu (x) = P_{max} \left(1-e^{-\mu(\Theta+kx)}\right)$$

where:
$\Delta P_\mu (x)$ = loss of prestressing force from 0 up to x distance [kN]
$x$ = distance from the stressing point [m]
$P_{max}$ = force at the stressing end [kN]
$\mu$ = friction coefficient between strands and ducts [1/rad]
$\theta$ = sum of the angular deviation from 0 up to x distance, irrespective of direction or sign [rad]
$k$ = not intentional angular deviation inside tendons, wobble coefficient [rad/m]
Values for friction coefficient $\mu$ are between 0,18 and 0,22 while k values are between 0,005 and 0,01.
Recommended values for calculation are $\mu$ = 0,19 [1/rad] and k = 0,005 [rad/m].

### Basic elongation evaluation factors

The elongation of the tendon, under the action of one or two jacks, is determined by the designer and verified at the construction site. The elongation calculation must clearly specify the theoretical elongation calculation and the corrective elements taken into account. It must be possible, with great accuracy, to establish the relationship between calculated elongation and measured elongation. In fact, secondary terms should be added to the elongation of the tendon along its length up to the anchorages, to obtain the real elongation.
Real elongation measured at the construction site is thus defined as the sum of the following elements:

$$DL_o = DL_a + DL_b + DL_r + DL_v$$

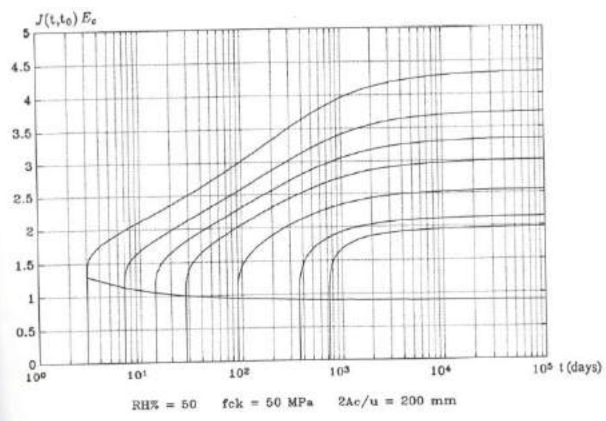
- DLa: theoretical elongation of the strand, calculated on the basis of the length between the anchorages, at one or two ends, depending on whether one or two jacks are used.
- DLb: concrete elastic shortening. This bit of information, since it is extremely small, is usually neglected.
- DLr: accumulation of deformations of the anchorage devices, for the deflection of the anchorages into the concrete and for the draw-in of the wedges.
- DLv: draw-in of the strand wedges within the jack and jack deformations.

# CREEP EFFECTS

### Creep law

$$\varepsilon = \sigma_0/J(t,t_0) \quad ; \quad \varepsilon = \int_0^t d\sigma(t')J(t,t')$$

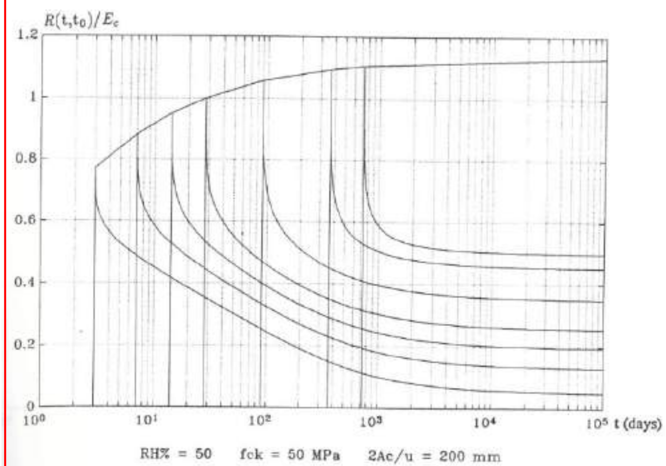
$$J(t,t_0) = \frac{1}{E(t_0)}\big(1+\varphi(t,t_0)\big)$$




### Relaxation law

$$\sigma = \varepsilon_0 R(t,t_0) \quad ; \quad \sigma = \int_0^t d\varepsilon\,(t')R(t,t')$$

$$R(t,t_0) = E(t_0)\big(1-\rho(t,t_0)\big)$$




### Convolution Integral

$$\int_0^t \frac{\partial R(t',t_0)}{\partial t'} J(t,t')dt' = 1$$

### Algebraic form of creep law

$$\varepsilon = \frac{\sigma}{E}(1+\chi\varphi) + \frac{\sigma_0}{E}(1-\chi)\varphi$$

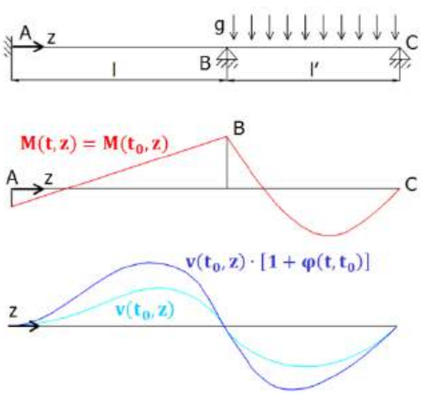
### Algebraic form of relaxation law

$$\sigma = \frac{E\varepsilon}{1+\chi\varphi} - \sigma_0 \cdot \frac{(1-\chi)\varphi}{1+\chi\varphi}$$

### First Theorem

$$S = S_e$$

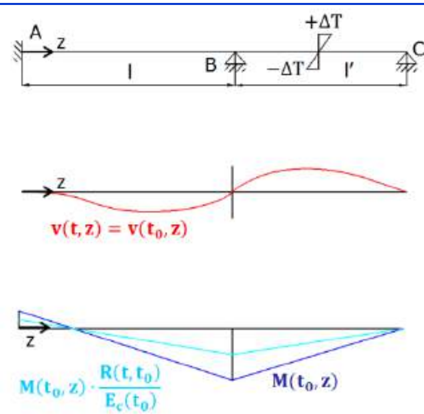
$$\underline{\sigma} = \underline{\sigma}_e$$
$$\underline{\varepsilon} = \underline{\varepsilon}_e(1+\varphi)$$
$$s = s_e(1+\varphi)$$




### Second Theorem

$$s = s_e$$

$$\underline{\varepsilon} = \underline{\varepsilon}_e$$
$$\underline{\sigma} = \underline{\sigma}_e(1-\rho)$$
$$S = S_e(1-\rho)$$



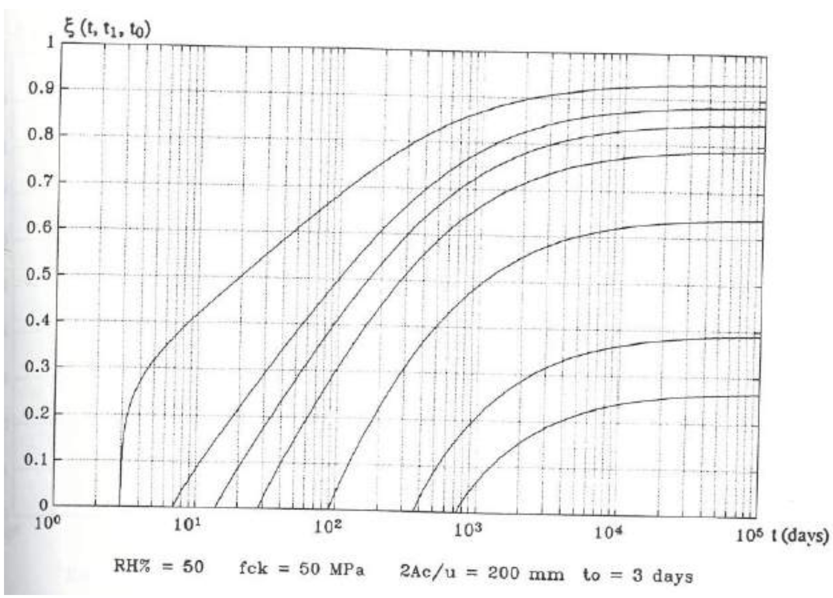
# CREEP EFFECTS

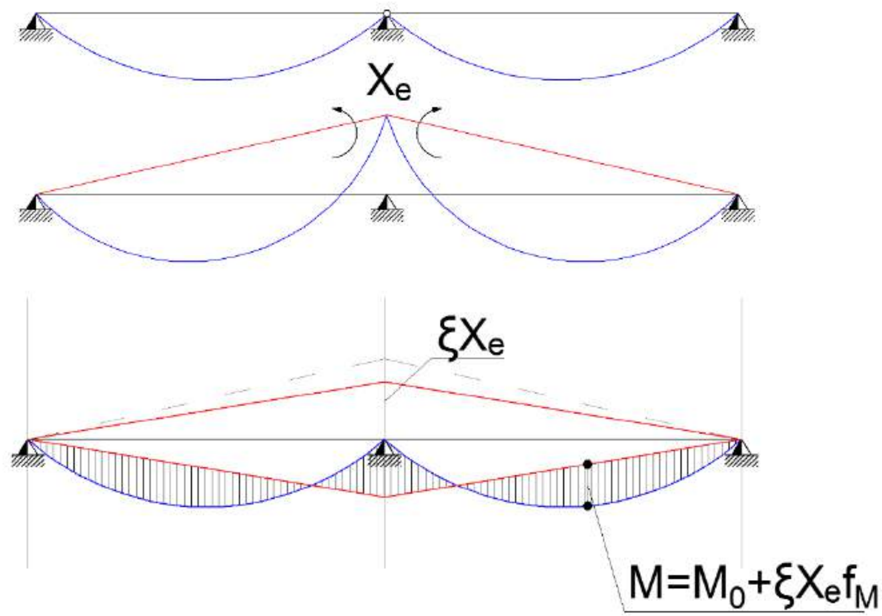
## Delayed Restratint

$$\varepsilon = \int_{t_0^*}^{t} dX\delta_{11}J = -\delta_{10}(\varphi - \varphi^*)$$

$$X = X_e\xi$$

$$M = M_0 + \xi \cdot X_e \cdot f_{M1}$$




Simply supported $\quad M(z) = \frac{4qL^2}{8}\left(-\frac{z^2}{L^2} + \frac{z}{L}\right)$

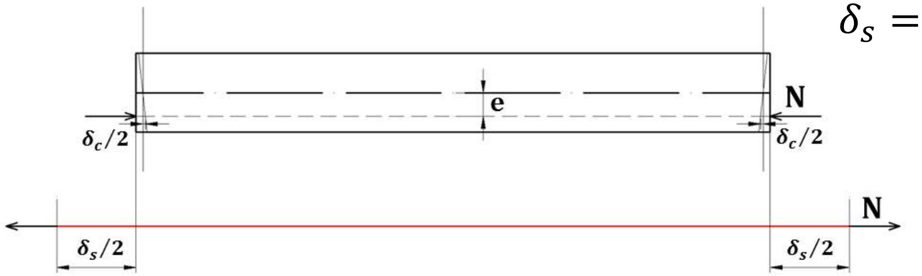
Iperstatic effect $\quad M(z) = -\xi\frac{qL^2}{8}\frac{z}{L}$

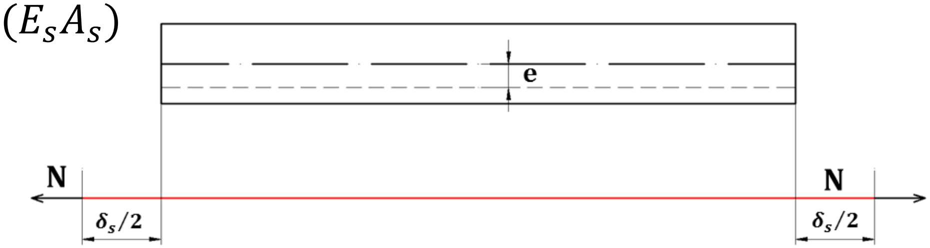
Resultant moment $\quad \frac{M(z)}{\frac{qL^2}{8}} = 4(-\zeta^2 + \zeta) - \xi\zeta \quad ; \quad \xi=0.7$

- **Prestressing force over time:**

$$\delta_c = \frac{1}{(E_c A_c)} + \frac{e^2}{(E_c I_c)} = \frac{1}{(E_c A_c)}(1 + \frac{e^2}{r^2})$$

$$\delta_s = \frac{1}{(E_s A_s)}$$




$$[\delta_c(1+\chi\varphi) + \delta_s]N + \delta_c\varphi(1-\chi)N_0 = \begin{matrix} N_p\delta_s & \text{Pre-tensioned elements} \\ N_p(\delta_c+\delta_s) & \text{Post-tensioned elements} \end{matrix}$$

$$\omega = \frac{\delta_c}{(\delta_c+\delta_s)} = \frac{U_c}{(U_c+U_s)}$$

$$N = N_p(1-\omega)\left[1 - \frac{\omega\varphi}{1+\chi\omega\varphi}\right] \quad \text{Pre-tensioned elements} \quad (1)$$

$$N = N_p\left[1 - \frac{\omega\varphi}{1+\chi\omega\varphi}\right] \quad \text{Post-tensioned elements} \quad (2)$$

$$\omega = \frac{\lambda\alpha_e\rho_{sp}}{1+\lambda\alpha_e\rho_{sp}}; \quad \alpha_e = \frac{E_c}{E_s}; \quad \lambda = \left(1 + \frac{e^2}{r^2}\right); \quad \rho_{sp} = \frac{A_{sp}}{A_c} = \frac{\omega}{(1-\omega)\alpha_e\lambda} \quad (3)$$

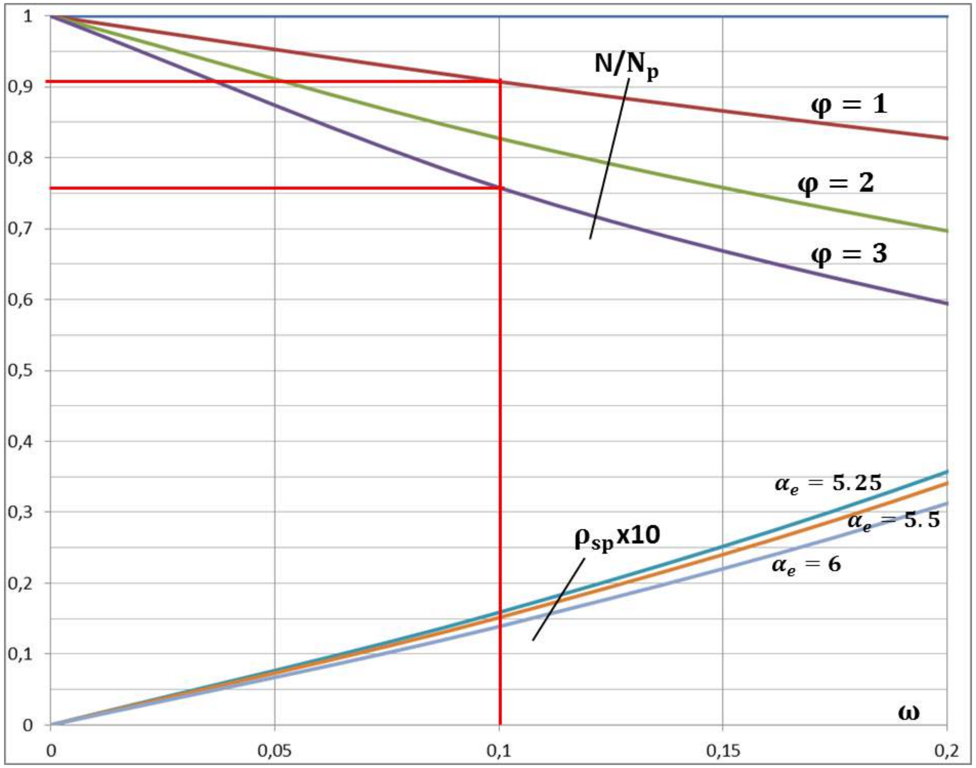
- **For integral prestressing, at t=∞, it results:**

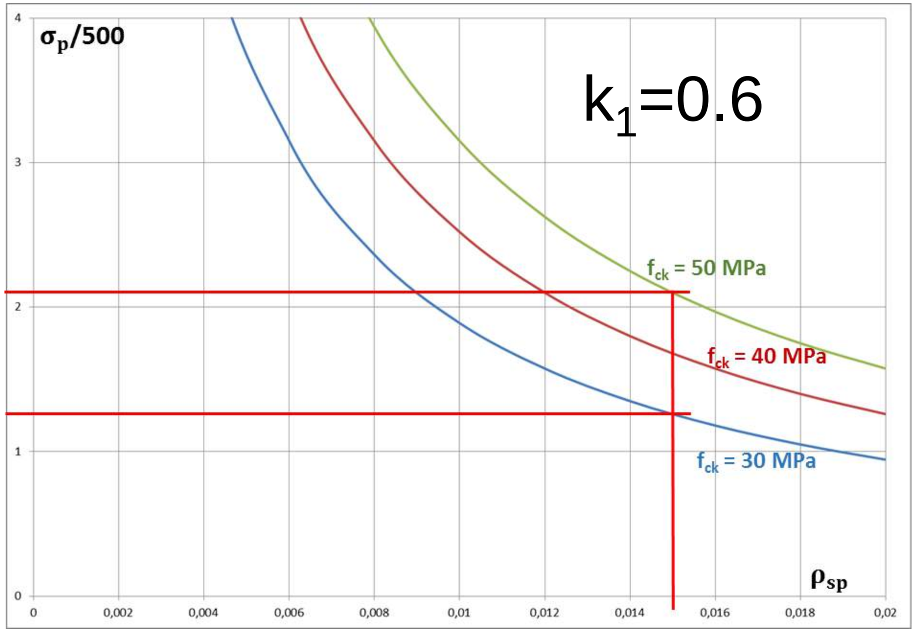
$$\frac{N_p}{A_c}\left[1 + \frac{e_n^{(i)}}{\left|e_n^{(s)}\right|}\right] = -\frac{M}{W_i} = -k_1 f_{ck}; \quad N_p = -\left[\frac{k_1 f_{ck} A |y_s|}{h}\right] \quad \sigma_p = -\left[\frac{k_1 f_{ck} A |y_s|}{h\varphi_{sp}}\right]$$

- **For the rectangular section, it can be written:**

$$e_n^{(i)} = \left|e_n^{(s)}\right|; \quad \lambda = \frac{4}{3}; \quad \rho_{sp} = \frac{0.75\omega}{(1-\omega)\alpha_e} \qquad \left|N_p\right| = k_1 f_{ck}\frac{A_c}{2} = \sigma_p A_s \qquad \sigma_p = \frac{k_1 f_{ck}}{2\rho_{sp}} \quad (4)$$

# THE VARIATION OF THE PRESTRESSING FORCE



# THE TOTAL VARIATION

$$\Delta N = N_P\left[1+\alpha_{sh}\frac{\beta_{sh}}{\varphi}+\alpha_R(1-\omega)-\alpha_g(1-\omega)\right]g(\varphi,\omega)$$

$$\alpha_{sh}\frac{(EA)_c}{N_p}\cdot\frac{\varepsilon_{sh}^{(\infty)}}{\lambda}\qquad \beta_{sh}=\frac{\varepsilon_{sh}(t)}{\varepsilon_{sh}(\infty)}\qquad \alpha_R=\frac{\Delta N_r^0}{N_p}$$

$$\alpha_g=\frac{M_g}{\lambda e N_p}\qquad \lambda=1+\frac{e^2}{r^2}\qquad g(\varphi,\omega)=\frac{\omega\varphi}{1+\chi\omega\varphi}$$

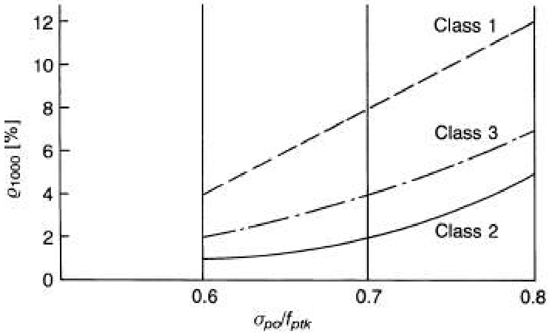
- The design calculations for the losses due to relaxation of the prestressing steel should be based on the value of $\rho_{1000}$, the relaxation loss (in %) at 1000 hours after tensioning and at a mean temperature of 20 °C, with $\sigma_{pi}$ / $f_{pk}$ = 0.7.

- Model Code 90

Three clauses of relaxation are defined:

1. Normal relaxation characteristics for wires and strands
2. Imposed relaxation characteristics for wires and strands
3. Relaxation characteristic for bars

For the above mentioned clauses the relaxation losses $\rho_{1000}$ % at 1000 h, varying the level of initial prestressing $\sigma_{p0}$ / $f_{ptk}$ are given in the next figure:




A time variation up to 1000 h is given in the following table:

| Time in hours | 1 | 5 | 20 | 100 | 200 | 500 | 1000 |
|---|---|---|---|---|---|---|---|
| Relaxation losses as percentage of losses in 1000 hours | 25 | 45 | 55 | 70 | 80 | 90 | 100 |

For an extimate of the relaxation up to 30 years the following formula can be applied: $\rho(t)=\rho_{1000}\left(\frac{t}{1000}\right)^k$ $\qquad k=log\left(\frac{\rho_{1000}}{\rho_{100}}\right)$

# THE TOTAL VARIATION

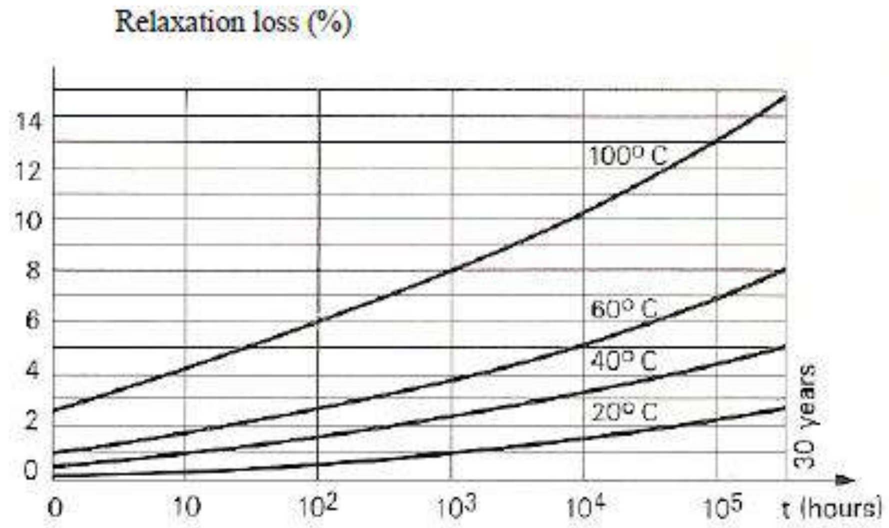
• EC2

This document defines the values of $\rho_{1000}$ in the following way:

- Class 1: $\rho_{1000}$ = 8%
- Class 2: $\rho_{1000}$ = 2.5%
- Class 3: $\rho_{1000}$ = 4%

connected to a ratio $\sigma_{pi}$ / $f_{pk}$ = 0.7

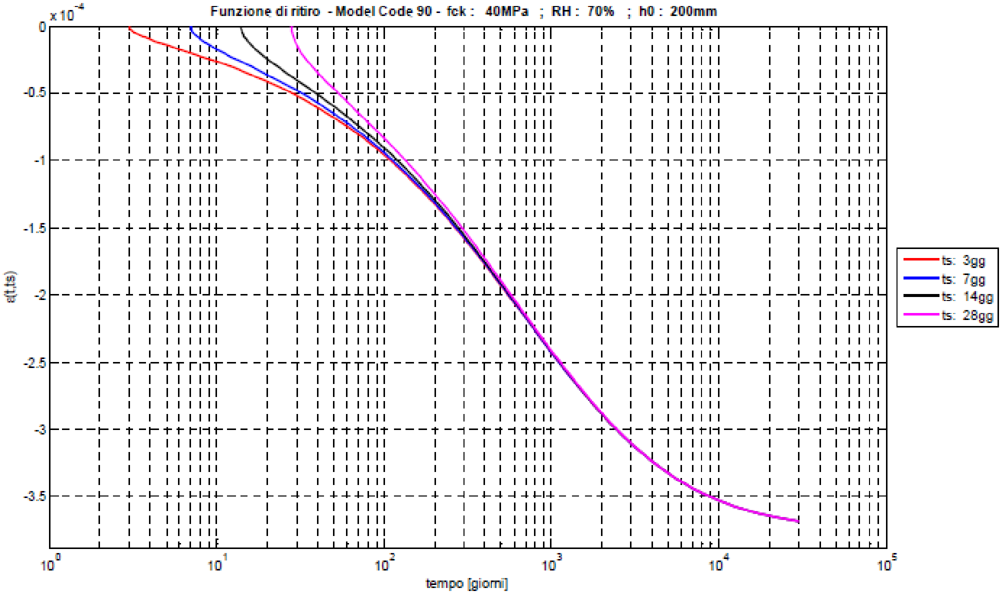
Introducing the time-variation of the relaxation losses for different levels of temperature:

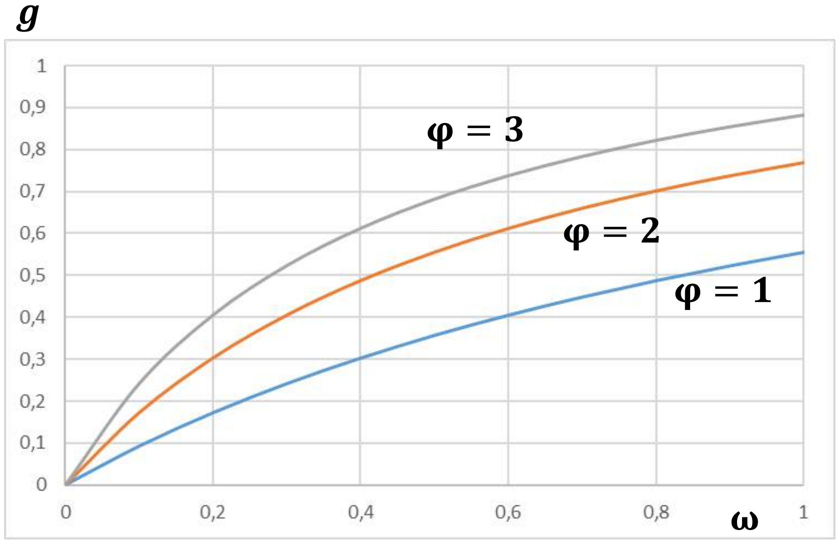


# THE TOTAL VARIATION

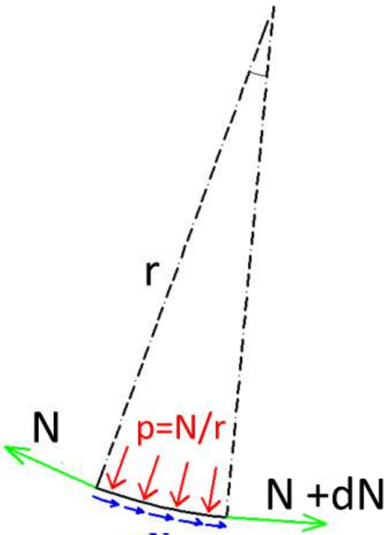
$$\varepsilon_{sh}(t) = \beta_{sh} \cdot \varepsilon_{sh}(\infty)$$

$$g(\varphi, \omega) = \frac{\omega\varphi}{1 + \chi\omega\varphi}$$



# PRESTRESSING LOSS DUE TO FRICTION




$$t = \frac{\mu N}{r}$$

$$\frac{dN}{d\vartheta} + \mu N = 0$$

$$N = N_0 e^{-\mu\vartheta}$$

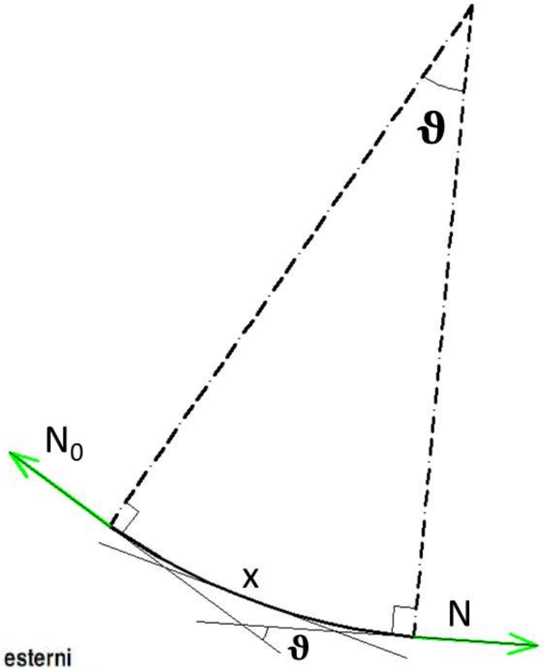
to the exponential part, the term

$$c = -\mu k x$$

has to be added, so obtaining

$$N = N_0 e^{-\mu(\vartheta + kx)}$$

$$0.005 \leq k \leq 0.01 [rad/m]$$




prospetto 5.1 **Coefficienti di attrito $\mu$ di armature post-tese in cavi interni e armature non aderenti in cavi esterni**

| | Armature interne[1] | Armature esterne non aderenti | | | |
|---|---|---|---|---|---|
| | | Condotto di acciaio/non lubrificato | condotto HDPE/non lubrificato | Condotto di acciaio/lubrificato | Condotto HDPE/ lubrificato |
| Filo laminato a freddo | 0,17 | 0,25 | 0,14 | 0,18 | 0,12 |
| Trefolo | 0,19 | 0,24 | 0,12 | 0,16 | 0,10 |
| Barra deformata | 0,65 | - | - | - | - |
| Barra liscia rotonda | 0,33 | - | - | - | - |

1) Per armature che riempiono all'incirca metà del condotto.

Nota HDPE - Polietilene ad alta densità.

- The equilibrium of the cable:

$$N_P \sim N_{0p}$$

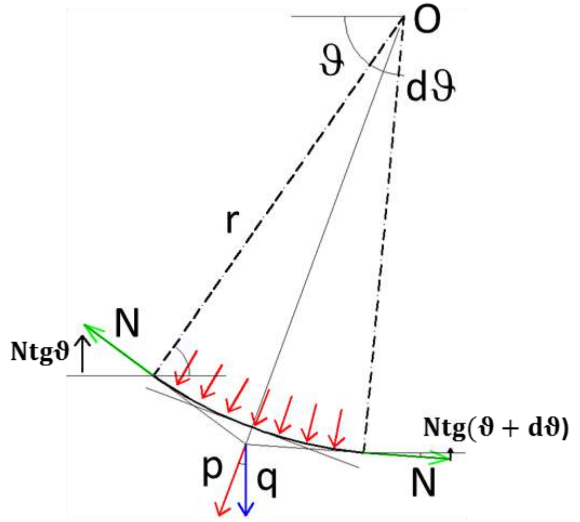
$$V_{1p} = N_p tg\alpha = N_p \cdot e'(0)$$

- The funicular equilibrium of cables with small curvature:

$$2N\left(\frac{d\vartheta}{2}\right) = prd\vartheta \qquad p = \frac{N}{r} \qquad \frac{d}{d\vartheta}(Ntg\vartheta)d\vartheta = qrd\vartheta$$

$$tg\vartheta \cong \vartheta \rightarrow prd\vartheta = qrd\vartheta \rightarrow p \cong q$$

$$\frac{1}{r} = -e''(z) \qquad q(z) = -Ne''(z)$$

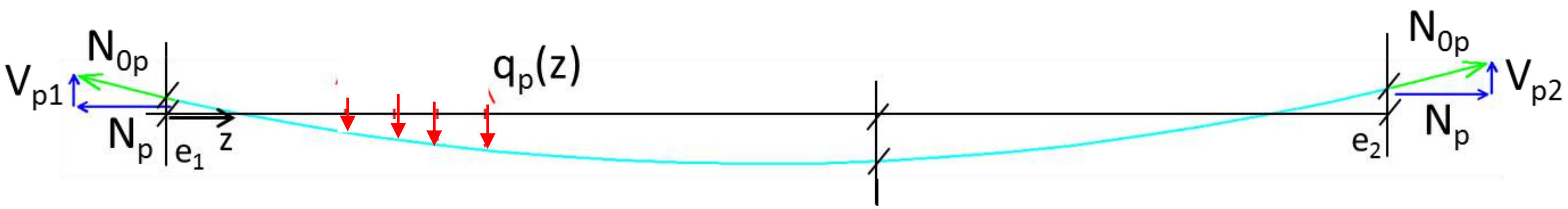



$$M(z) = N_p \cdot [e_1 - e(z)] + V_{1p} \cdot z - \int_0^z N_p e''(\bar{z})(\bar{z} - z)d\bar{z} =$$

$$= N_p \cdot [e_1 - e(z)] + N_p \cdot e'(0) \cdot z - \left[N_p e'(\bar{z})(\bar{z} - z)\right]_0^z - \int_0^z N_p e''(\bar{z})d\bar{z} =$$

$$= N_p \cdot [e_1 - e(z)] + N_p \cdot e'(0) \cdot z - N_p \cdot e'(0) \cdot z + N_p \cdot [e(z) - e_1] = 0$$

$$V(z) = V_{1p} - \int_0^z -N_p e''(\bar{z})d\bar{z} = N_p \cdot e'(0) + N_p \cdot [e'(z) - e'(0)] = N_p \cdot e'(z)$$



# REDUNDANT STRUCTURES: EQUIVALENT LOADS

- The equilibrium of the beam:


$$\mathbf{M(z) = N_p \cdot e_1 + V_{1p} \cdot z + \int_0^z -N_p e''(\bar{z})(\bar{z} - z) d\bar{z} =}$$

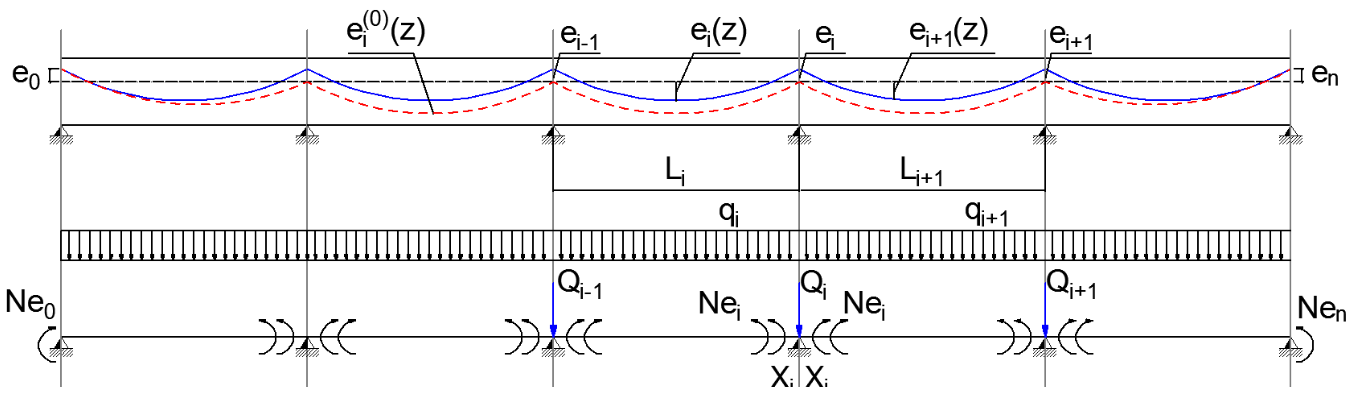
$$\mathbf{= N_p \cdot e_1 + N_p \cdot e'(0) \cdot z - N_p \cdot e'(0) \cdot z + N_p \cdot [e(z) - e_1] = N_p \cdot e(z)}$$

$$\mathbf{V(z) = M'(z) = N_p \cdot e'(z)}$$

$$\mathbf{q = -N_p \cdot e''(z)}$$

The system of loads $N_p$; $V_{1p}$; $N_p$; $V_{2p}$; $q_p(z)$; $N_p e_1$; $N_p e_2$ is self balanced and balances the one due to external loads, thus improving structural response.

# THE LINEAR TRANSFORMATION THEOREM




$$e(z) = e^0(z) + \underline{a}^T\underline{e} + e_M(z)$$

$$q(z) = -Ne^{0''}(z)$$

$$M_0^0(z) = Ne_M(z)$$

$$e_M(z) = e_0\left(1-\frac{z}{L}\right) \quad \text{I span}$$

$$e_M(z) = 0 \quad \text{II} \rightarrow (n-1) \text{ span}$$

$$e_M(z) = e_n(z)\frac{z}{L} \quad \text{n span}$$

$$M(z) = M_q^0(z) + M_0^0(z) + \underline{a}^T(z)\cdot(\underline{X} + N\underline{e})$$

$$\delta M(z) = \underline{a}^T\delta\underline{X}$$

$$0 = \delta\underline{X}^T\int_s \underline{a}\frac{M(z)}{EI}dz = \delta\underline{X}^T\int_s \underline{a}\,\underline{a}^T(\underline{X}+N\underline{e})\frac{dz}{EI} + \int_s \underline{a}M_q^0(z)\frac{dz}{EI} + \int_s \underline{a}M_0^0(z)\frac{dz}{EI}$$

$$\int_s \underline{a}\,\underline{a}^T\frac{dz}{EI} = \underline{F}_e; \qquad \int_s \underline{a}M_q^0(z)\frac{dz}{EI} = \underline{\delta}_{0q}; \qquad \int_s \underline{a}M_0^0(z)\frac{dz}{EI} = \underline{\delta}_{0M}$$

$$\underline{F}_e(\underline{X}+N\underline{e}) + \underline{\delta}_{0q} + \underline{\delta}_{0M} = 0 \rightarrow \underline{X} = -N\underline{e} + \underline{X}_q + \underline{X}_M\,;\ \underline{X}_q = -\underline{F}_e^{-1}\underline{\delta}_{0q}\,;\ \underline{X}_M = -\underline{F}_e^{-1}\underline{\delta}_{0M}$$

$$\boxed{\begin{gathered} M(z) = M_q^0(z) + M_0^0(z) + \underline{a}^T(z)\cdot(\underline{X}_q + \underline{X}_M) = M_q(z) + M_0(z) \\ M_q(z) = M_q^0(z) + \underline{a}^T(z)\underline{X}_q \\ M_0(z) = M_0^0(z) + \underline{a}^T(z)\underline{X}_M \end{gathered}}$$

If in a continuous beam we change the eccentricity of the cable at the internal supports maintaining constant the eccentricity of the two extreme supports and the curved form of the cable, the bending moment in the beam remains unchanged and results:

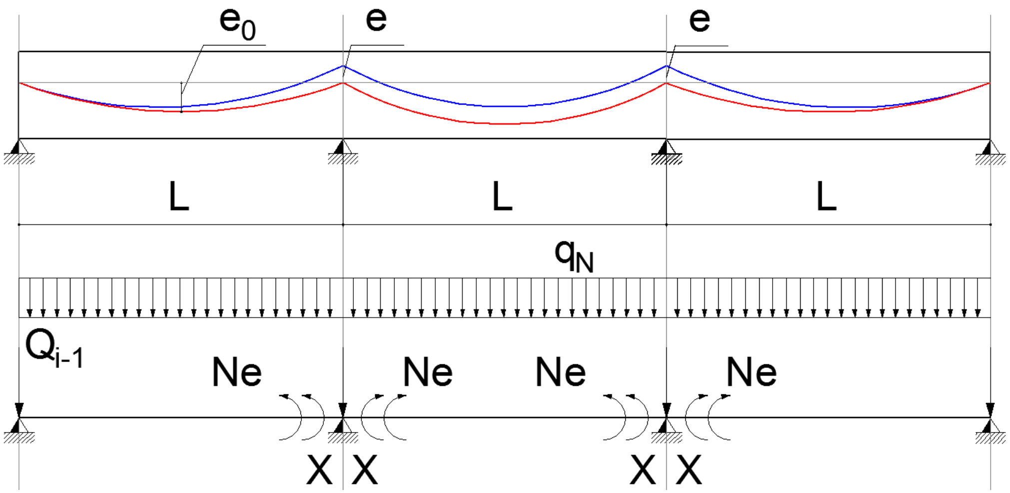
$$M(z) = M_q(z) + M_0(z)$$

where $M_q(z)$, $M_0(z)$ are the bending moments due to the distributed loads and to the moments at the extreme points. In particular, if:

$$N\underline{e} = \underline{X}_q + \underline{X}_M \rightarrow e_i = \frac{X_{qi}+X_{Mi}}{N}$$

the bending moment in the beam coincides with the one due to the cable eccentricity, i.e $M(z) = Ne(z)$, no reactions arise in the continuity supports and the cable is called concordant.

# THE LINEAR TRANSFORMATION THEOREM




$$e(z) = 4e^0\left(-\frac{z^2}{L^2}+\frac{z}{L}\right)$$

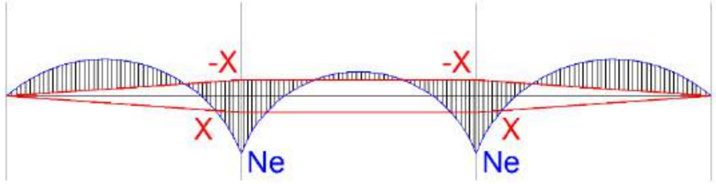
$$e_1(z) = 4e_0\left(-\frac{z^2}{L^2}+\frac{z}{L}\right)+e\frac{z}{L} \qquad q_N = -N\cdot\left(-\frac{8e_0}{L^2}\right) = \frac{8Ne_0}{L^2}$$

$$e_2(z) = 4e_0\left(-\frac{z^2}{L^2}+\frac{z}{L}\right)+e$$

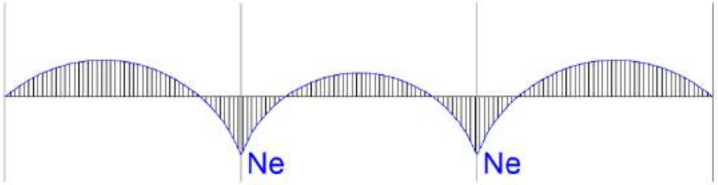
$$e_3(z) = 4e_0\left(-\frac{z^2}{L^2}+\frac{z}{L}\right)+e\left(1-\frac{z}{L}\right)$$

$$X = -\frac{qL^2}{10} - Ne = -\frac{qL^2}{10} - \frac{qL^2}{8e_0}\cdot e \qquad N = \frac{q_N L^2}{8e_0}$$

$$\text{If}\quad e = -e_0 \quad\rightarrow\quad X = -\frac{qL^2}{10}+\frac{qL^2}{8} \quad\rightarrow\quad M = Ne + X\frac{z}{L} = -\frac{qL^2}{8}e(z)+\frac{qL^2}{40}\frac{z}{L}$$

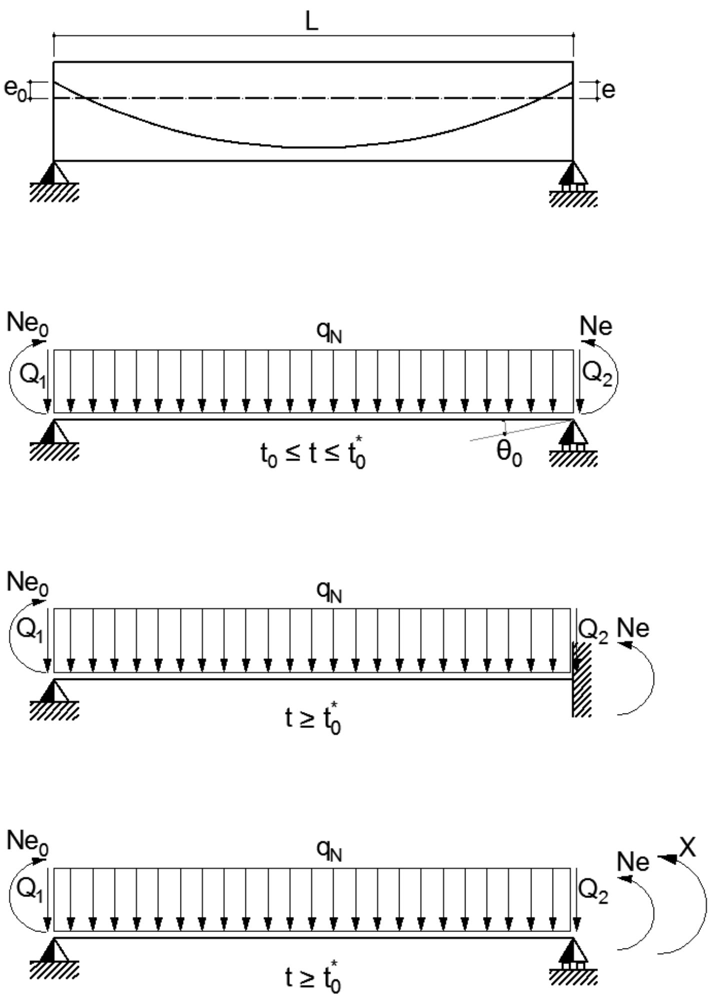



$$\text{If}\quad \frac{e}{e_0} = \frac{8}{10} \quad\rightarrow\quad X = 0 \quad\rightarrow\quad M = Ne$$



# THE DELAYED RESTRAINT

## a) ROTATIONAL DELAYED RESTRAINT




$$\Delta\theta_0 = \left(\theta_0^{\ (q_N)} + \theta_0^{\ (Ne_0)} + Ne\delta_{11}\right)\left(\varphi(t,t_0) - \varphi(t_0^*,t_0)\right)$$

$$\Delta\theta_X = \int_{t_0^*}^{t} \delta_{11} dX(t') \cdot E(t_0) \cdot J(t,t')$$

Compatibility:

$$\Delta\theta_0 + \Delta\theta_X = 0$$

$$\int_{t_0^*}^{t} dX(t') \cdot E(t_0) \cdot J(t,t') = \left(-\frac{\theta_0^{\ (q_N)}}{\delta_{11}} - \frac{\theta_0^{\ (Ne_0)}}{\delta_{11}} - Ne\right) \cdot \left(\varphi(t,t_0) - \varphi(t_0^*,t_0)\right)$$

$$X = \left(X_e^{(q_N)} + X_e^{\ (Ne_0)} - Ne\right) \cdot \int_{t_0^*}^{t} \frac{\partial\varphi(t',t_0)}{\partial t'} \cdot \frac{R(t,t')}{E(t_0)} dt'$$

Indicating by:

$$\xi = \int_{t_0^*}^{t} \frac{\partial\varphi(t',t_0)}{\partial t'} \cdot \frac{R(t,t')}{E(t_0)} dt' \quad \text{appoximately} \quad \xi = \frac{\varphi(t,t_0) - \varphi(t_0^*,t_0)}{1 + \chi(t,t_0^*)\varphi(t,t_0^*)}$$

The bending moment so becomes:

$$M = M_0^{(q_N)}(z) + M_0^{(Ne_0)}(z) + X_e^{(q_N)} f_{M_2}(z) \cdot \xi + X_e^{(Ne_0)} f_{M_2}(z) \cdot \xi + Ne f_{M_2}(z) \cdot (1-\xi)$$

$$f_{M_2}(z) = z/L$$

Remembering that:

$$M_0^{(q_N)}(z) + X_e^{(q_N)} f_{M_2}(z) = M_e^{(q_N)}(z) \ \rightarrow \ X_e^{(q_N)} f_{M_2}(z) = M_e^{(q_N)}(z) - M_0^{(q_N)}(z)$$

$$M_0^{(Ne_0)}(z) + X_e^{(Ne_0)} f_{M_2}(z) = M_e^{(Ne_0)}(z) \ \rightarrow X_e^{(Ne_0)} f_{M_2}(z) = M_e^{(Ne_0)}(z) - M_0^{(Ne_0)}(z)$$

We write:

$$M = M_0^{(q_N)}(z)(1-\xi) + M_e^{(q_N)}(z) \cdot \xi + M_0^{(Ne_0)}(z)(1-\xi) + M_e^{(Ne_0)}(z)\xi + Ne f_{M_2}(z) \cdot (1-\xi)$$

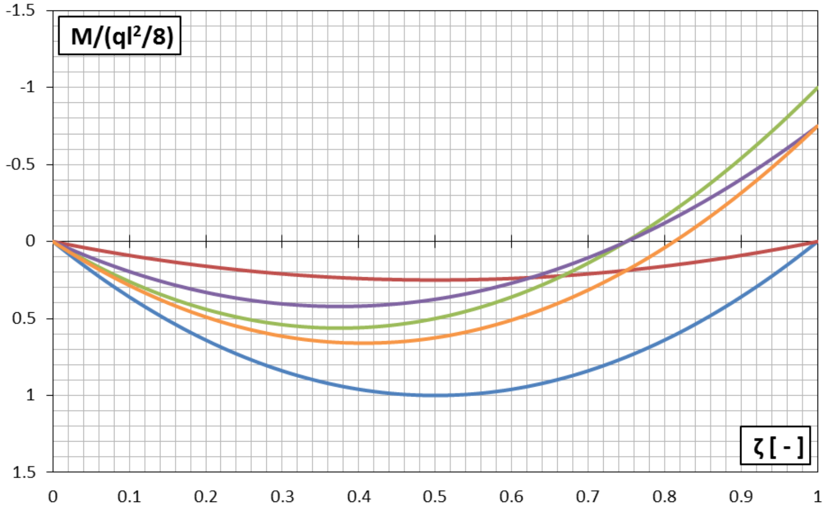
In the elastic stage $\xi=1$:

$$M = M_e^{(q_N)}(z) + M_e^{(Ne_0)}(z) \quad \text{(theorem of linear transformations)}$$

# THE DELAYED RESTRAINT

## a) ROTATIONAL DELAYED RESTRAINT

**Numerical Example $\xi = 0.75$ ; $0 \le \zeta \le 1$ ; $\zeta = z/l$**




(1) $$\frac{M_0^{(q_N)}}{\frac{ql^2}{8}} = 4(\zeta - \zeta^2)$$

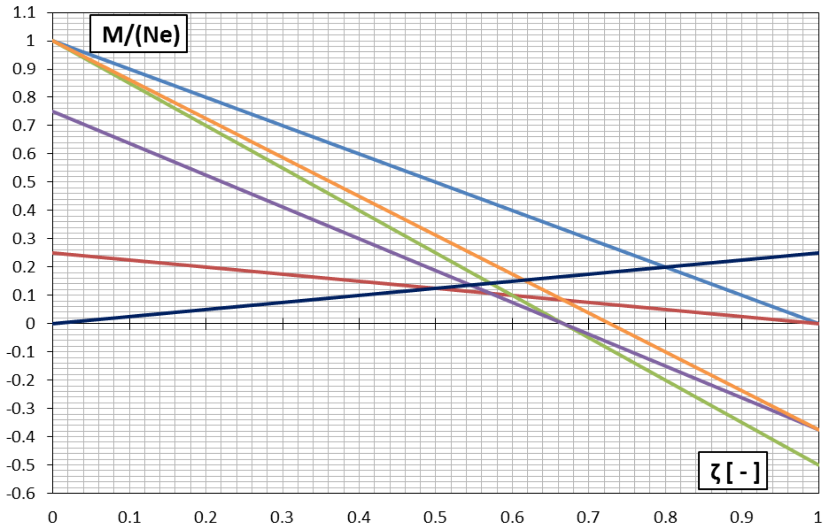
(2) $$\frac{M_0^{(q_N)}}{\frac{ql^2}{8}}(1-\xi) = 4(1-\xi)(\zeta - \zeta^2)$$

(3) $$\frac{M_e^{(q_N)}}{\frac{ql^2}{8}} = 4(\zeta - \zeta^2) - \zeta$$

(4) $$\frac{M_e^{(q_N)}}{\frac{ql^2}{8}}\xi = 4\xi(\zeta - \zeta^2) - \zeta\xi$$

(5) $$\frac{M_0^{(q_N)}}{\frac{ql^2}{8}}(1-\xi) + \frac{M_e^{(q_N)}}{\frac{ql^2}{8}}\xi = 4(\zeta - \zeta^2) - \zeta\xi$$

**Numerical Example $\xi = 0.75$ ; $0 \le \zeta \le 1$ ; $\zeta = z/l$**




(1) $$\frac{M_0^{(Ne_0)}}{Ne_0} = 1 - \zeta$$

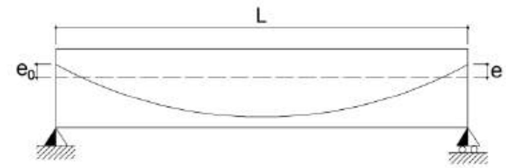
(2) $$\frac{M_0^{(Ne_0)}}{Ne_0}(1-\xi) = (1-\zeta)(1-\xi)$$

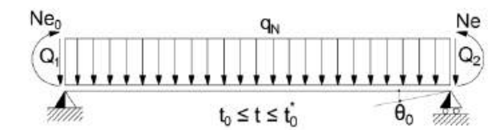
(3) $$\frac{M_e^{(Ne_0)}}{Ne_0} = 1 - \frac{3}{2}\zeta$$

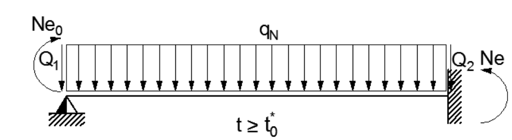
(4) $$\frac{M_e^{(Ne_0)}}{Ne_0}\xi = \xi\left(1 - \frac{3}{2}\zeta\right)$$

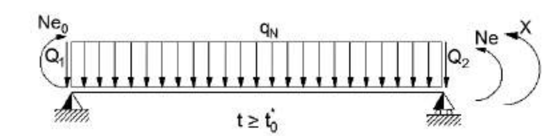
(5) $$\frac{M_0^{(Ne_0)}}{Ne_0}(1-\xi) + \frac{M_e^{(Ne_0)}}{Ne_0}\xi = 1 - \zeta\left(1 + \frac{\xi}{2}\right)$$

(6) $$\frac{Ne}{Ne}(1-\xi)\zeta = (1-\xi)\zeta$$



# THE DELAYED RESTRAINT

## b) TRANSLATIONAL ELASTIC RESTRAINT

### b.1) Curved parabolic cable

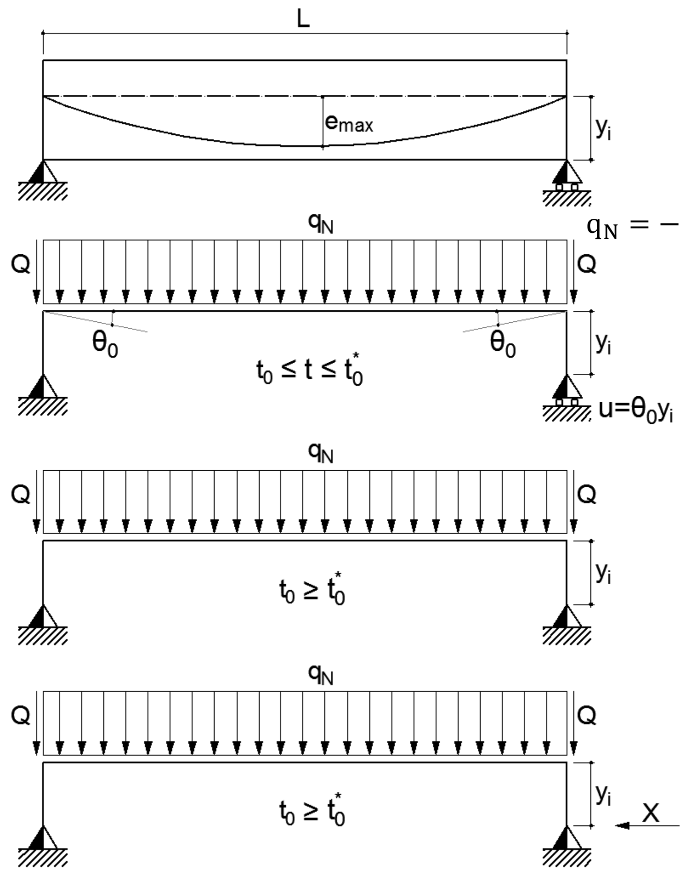
$$\int_{t_0^*}^{t} dX \cdot \delta_{11} \cdot E(t_0) = N_p \delta_{10} \cdot \left(\varphi(t,t_0) - \varphi(t_0^*,t_0)\right)$$

$$X = N_p \frac{\delta_{10}}{\delta_{11}} \int_{t_0^*}^{t} \frac{\partial(t',t_0)}{\partial t'} \cdot \frac{R(t,t')}{E(t_0)} dt' = N_p \frac{\delta_{10}}{\delta_{11}} \xi = N_p \cdot c \cdot \xi$$

$$\delta_{10} = \left(\frac{L}{EA} + \frac{2e_{max}y_i L}{3EI}\right) = \frac{L}{3EA}\frac{3r^2 + 2e_{max}y_i}{r^2}$$

$$\delta_{11} = \frac{L}{EA} + \frac{y_i^2 L}{EI} = \frac{L}{EA}\frac{r^2 + y_i^2}{r^2}$$

$$c = \frac{1}{3}\left(\frac{3r^2 + 2e_{max}y_i}{r^2 + y_i^2}\right)$$




$$q_N = -\frac{8N_p e_{max}}{L^2}$$

### b.2) Horizontal strands

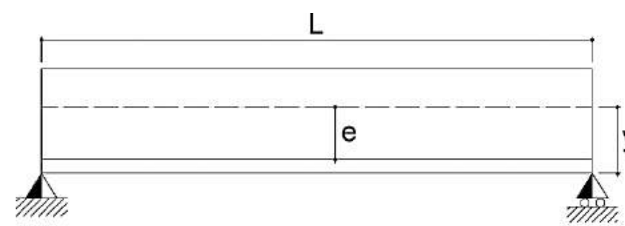
$$c = \frac{\delta_{10}}{\delta_{11}}$$

$$\delta_{10} = \frac{L}{EA} + \frac{e y_i L}{2EI} = \frac{L}{EA}\frac{r^2 + e y_i}{r^2}$$

$$\delta_{11} = \frac{L}{EA}\frac{r^2 + y_i^2}{r^2}$$

$$c = \frac{r^2 + e y_i}{r^2 + y_i^2}$$

$$X = N_p c \xi$$



# THE DELAYED RESTRAINT

In the two cases the components of action effects become:

$$N = N_P + X = N_P(1 - c\xi)$$

$$M = N_P \cdot e\,(1 - c\frac{y_i}{e}\xi)$$

For a rectangular cross section, it results $r^2 = h^2/12\,;\;\; y_i = 1/2h$ and assuming $e_{max} = e = 0.4h$ we derive:

$$c = \frac{\frac{1}{3}\left(\frac{3}{12} + 2 \cdot 0.4 \cdot 0.5\right)}{\frac{1}{12} + 0.5^2} = 0.65 \quad \text{(parabolic cable)}$$

$$c = \frac{\left(\frac{1}{12} + 0.4 \cdot 0.5\right)}{\frac{1}{12} + 0.5^2} = 0.85 \quad \text{(straight strand)}$$

Assuming $\varphi(t, t_0) = 2.5; \varphi(t_0^*, t_0) = 0.9; \varphi(t, t_0^*)_{t=\infty} = 2$ we derive the function:

$$\xi = \frac{2.5 - 0.9}{1 + 0.8 \cdot 2} = 0.615$$

So for the two cases we obtain:

Parabolic cable

$$N = N_P(1 - 0.65 \cdot 0.615) = 0.6N_p$$

$$M = N_p\left(1 - 0.72 \cdot 0.615 \cdot \frac{0.5}{0.4}\right)e_{max} = 0.446N_p e_{max}$$

Straight strand

$$N = N_P(1 - 0.85 \cdot 0.615) = 0.477N_p$$

$$M = N_p\left(1 - 0.88 \cdot 0.615 \cdot \frac{0.5}{0.4}\right)e = 0.323N_p e$$

# SEGMENTAL STRUCTURES

## a) Service Stage

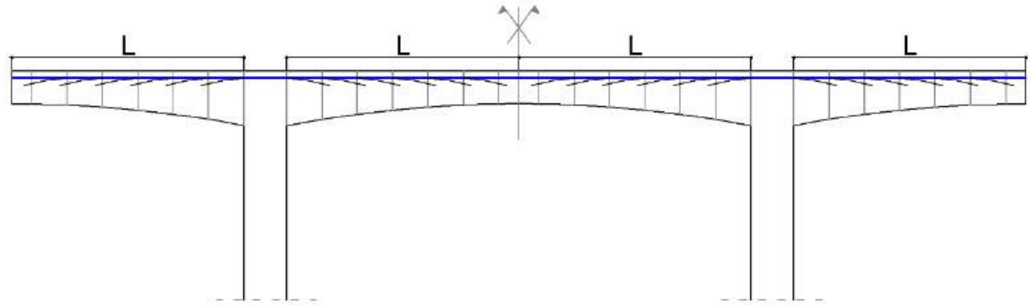
$$\frac{|N_p|}{A} + \frac{|N_P||e_p|}{W_s} \geq \frac{M_{Ek}}{W_s} \qquad \frac{W_s}{A_P} = e_{ni}$$

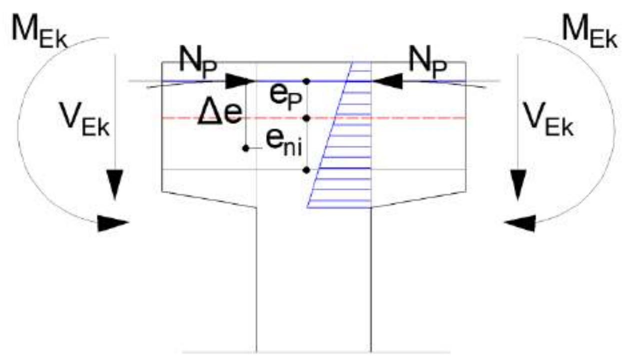
$$|N_p|[e_{ni} + |e_p|] \geq M_{Ek}$$

$$(e_{ni} + |e_p|) \geq \frac{M_{Ek}}{N_p} = \Delta e$$ decompression limit state

$$\mu|N_p| \geq V_{Ek}$$ shear limit state

$$0.5 \leq \mu \leq 0.9$$ (UNI EN 1992-1-1)




Zambezi river Bridge

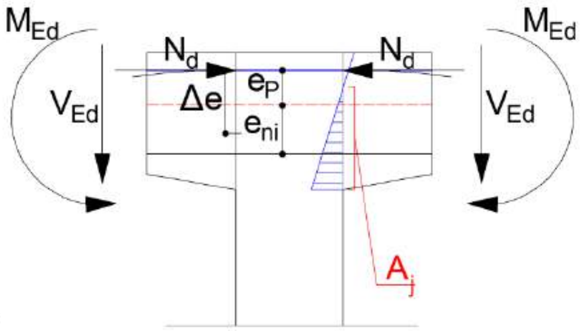
Moving scaffolding for balanced cantilever construction in Quintanilha Bridge

The bridge over the Ribeira Funda, Madeira Island, built by balanced cantilever (135 m main span) – Approaching the execution of the closure segment at mid-span

# SEGMENTAL STRUCTURES

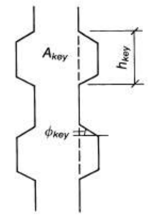
## b) Ultimate Limite State (CEB-FIB MODEL CODE 1990)

$$V_{Rd} = \min \begin{Bmatrix} \{\mu[A_s \cdot f_{yd}(1+\cot\alpha)\sin\alpha + N_d] + 0.1 A_k \cdot f_{cd}\}/\gamma_{Rd} \\ 0.3 \cdot A_j \cdot f_{cd} \end{Bmatrix}$$




where

- $\mu$ is 0.5 for smooth plane surfaces and 0.9 for rough or keyed surfaces
- $A_s, f_{yd}, \alpha$ are the cross-sectional area, design yield stress and inclination of the steel bars crossing the joint and being well anchored on both sides
- $N_d$ is the design (most unfavourable) normal force acting on the joint section (positive when compressive)
- $A_j$ is the cross-sectional area of the joint under compression
- $A_k, f_{cd}$ are the cross-sectional area of the portion of the joint keys interacting in the resistance, and the design strength of concrete in the considered joint portion ($A_k = 0$ for plain joints or when $A_k/A_j < 0.2$).




$A_j = b_j l_j$

$A_{key} = b_{key} h_{key} n$

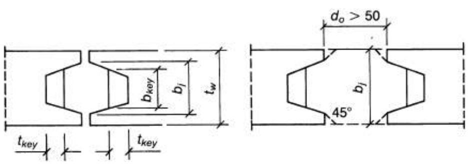
$l_j$ – joint length (e.g. storey height)

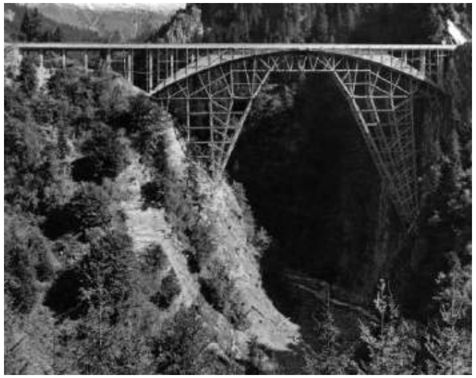
$n$ – number of keys at length $l_j$

$h_{key} \geqslant 50$ mm and $t_{key} \geqslant 10$ mm

$h_{key}/t_{key} < 8$

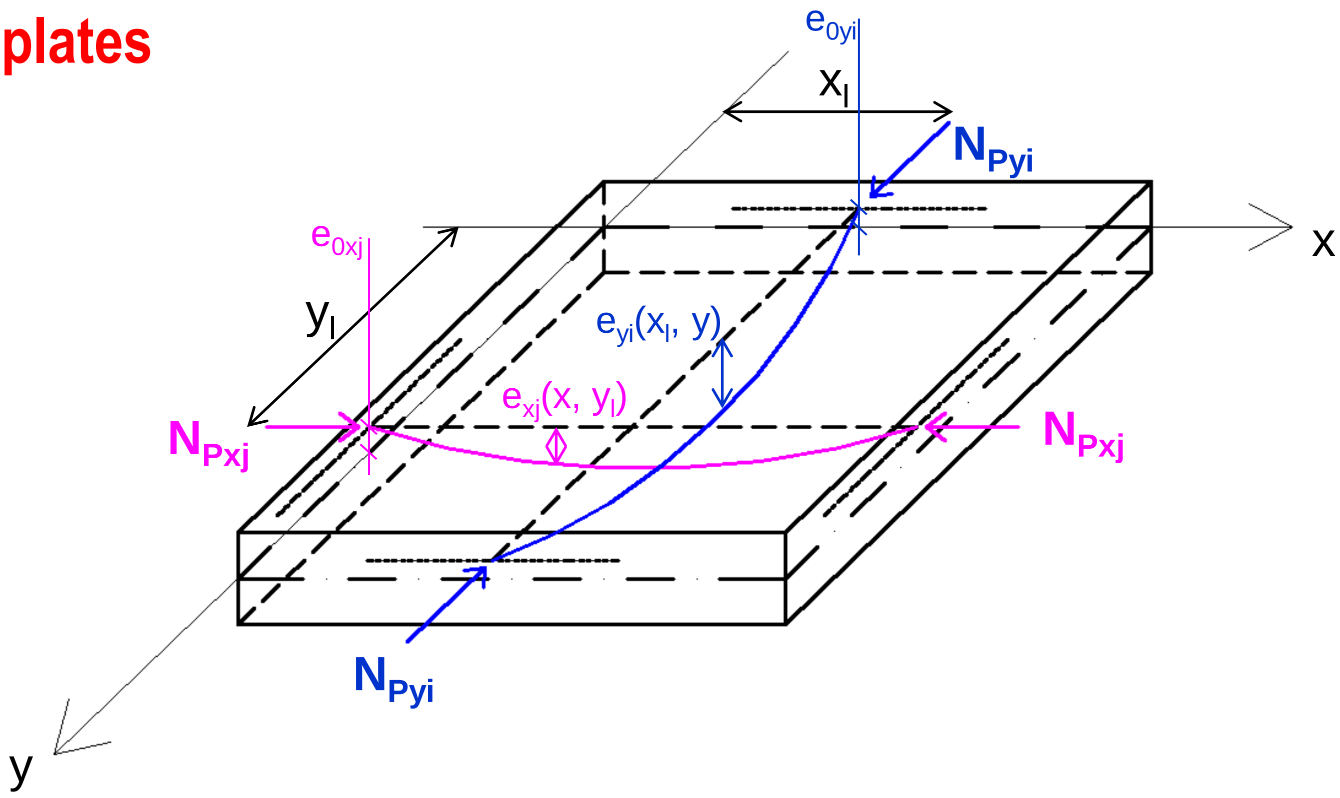
$\phi_{key} \leqslant 30°$



# THE PRESTRESSING OF BI-DIMENSIONAL ELEMENTS

- **Thin plates**

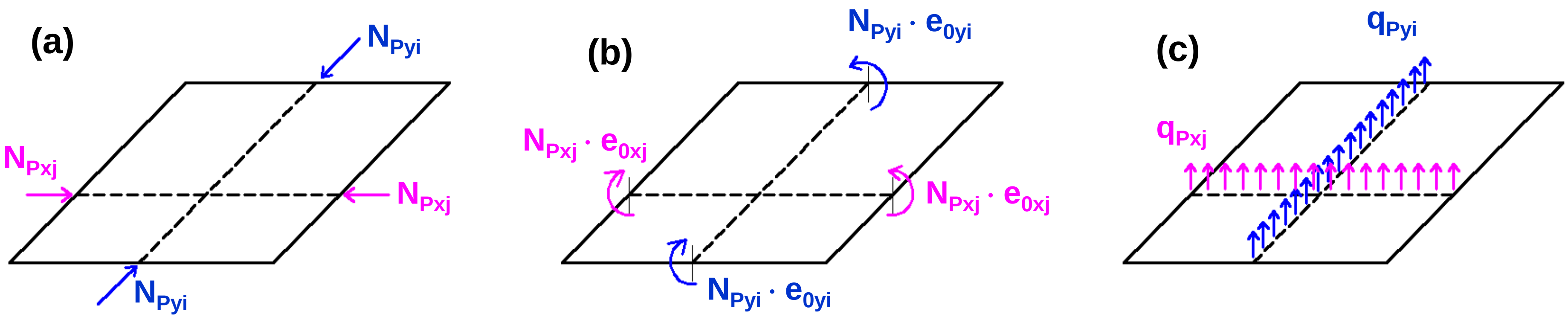



$$q_{Pyi} = -N_{pyi}\,\partial^2 e_{yi}(x_l, y)/\partial y^2$$

$$q_{Pxj} = -N_p\,\partial^2 e_{xi}(x, y_l)/\partial x^2$$

- **Thin plates**

The effects can be separated as:




a. Plate loaded by in-plane prestressing forces
b. Plate loaded by moments acting along the boundaries
c. Plate subjected to loads distributed along prestressing lines

*The problem is uncoupled if second order effects are negligible*

# THE PRESTRESSING OF BI-DIMENSIONAL ELEMENTS

**(1)**

In-plane forces

$$\nabla^4 \phi = 0$$

$$\frac{N_x}{t} = \frac{\partial^2 \phi}{\partial x^2}$$

$$\frac{N_y}{t} = \frac{\partial^2 \phi}{\partial y^2}$$

$$\frac{N_{xy}}{t} = \frac{\partial^2 \phi}{\partial x \partial y}$$

Out-of-plane forces

$$\nabla^4 w = 0$$

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right]$$

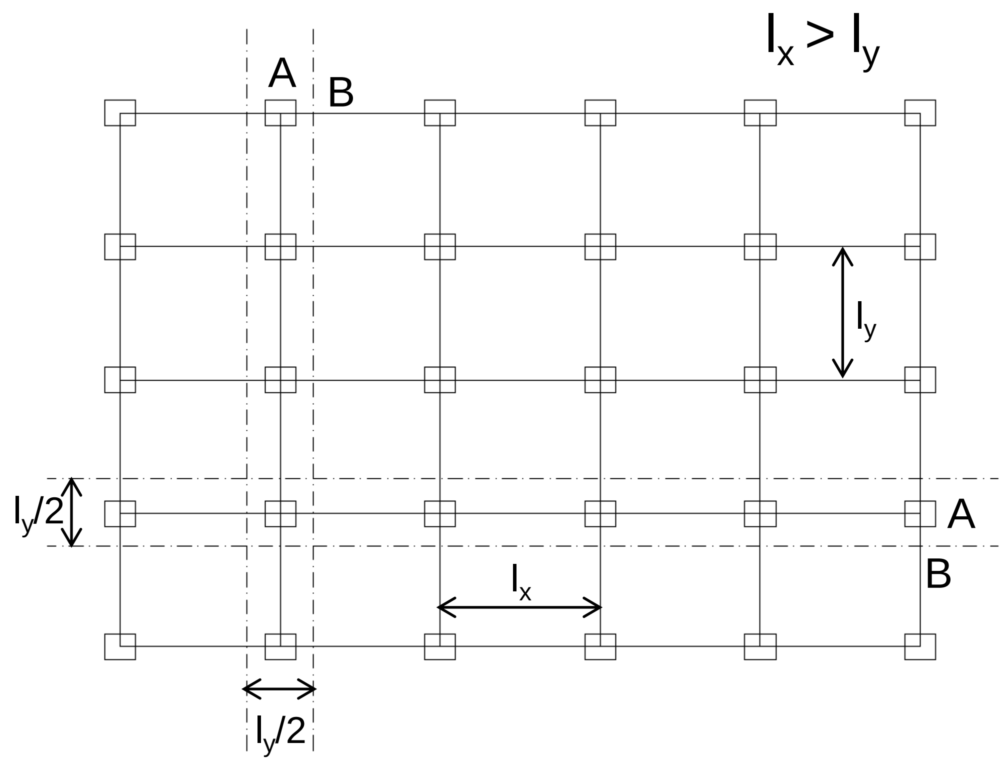
$$M_y = -D\left[\nu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right]$$

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$$

For the external loading, the governing equation is:

$$\nabla^{IV} w = \frac{q}{D} \qquad \textbf{(2)}$$

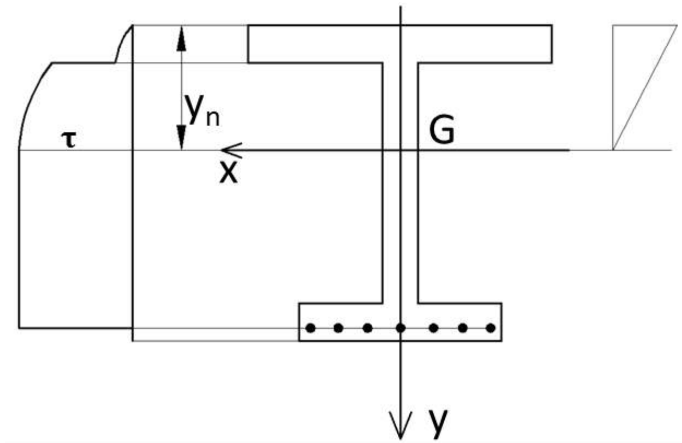
There is no affinity between (1) and (2), so that simplified analyses with the assumption of an equivalent beam must be carfully evaluated.

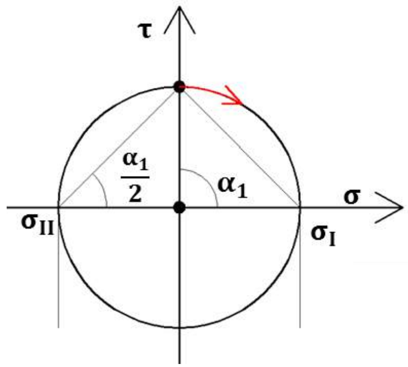


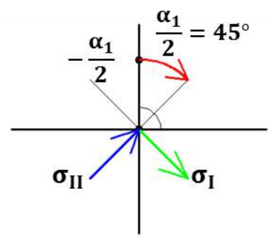

EC2
- $M^-$ (60%÷80%) / $M^-$ (50%÷70%) — Zona A
- $M^-$ (40%÷20%) / $M^-$ (50%÷30%) — Zona B

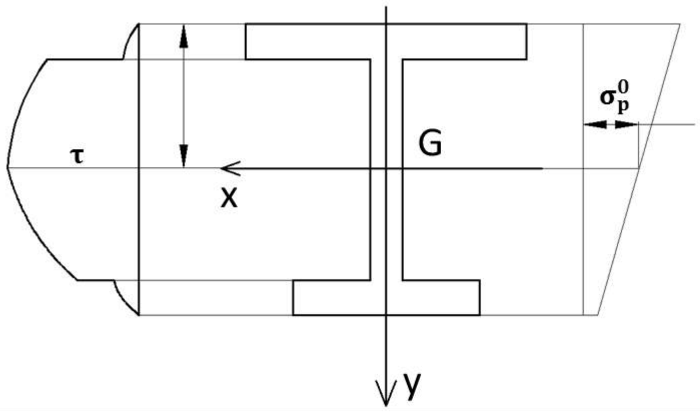
# SHEAR LIMIT STATE

- Service stage



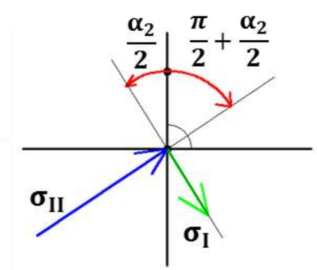



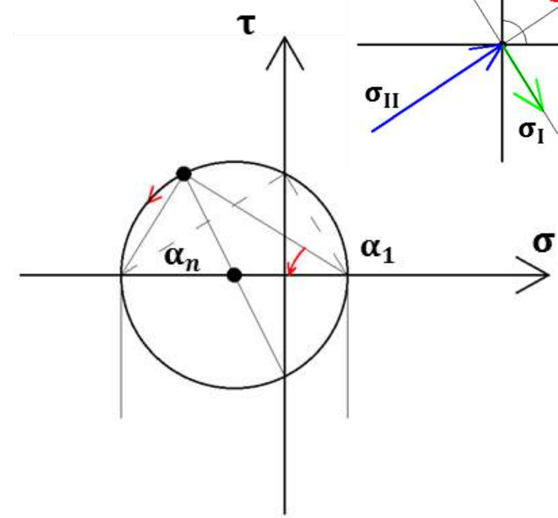

$$\sigma_I = \frac{\sigma_p^0}{2} + \frac{1}{2}\sqrt{\left(\sigma_p^0\right)^2 + 4\tau^2} > 0$$

$$\sigma_{II} = \frac{\sigma_p^0}{2} - \frac{1}{2}\sqrt{\left(\sigma_p^0\right)^2 + 4\tau^2} < 0$$

$$\mathrm{tg}\left(\frac{\alpha_2}{2}\right) = \frac{\tau}{\sigma_{II}}$$

$$\mathrm{tg}\left(\frac{\alpha_1}{2}\right) = \frac{\tau}{\sigma_I}$$



# SHEAR LIMIT STATE

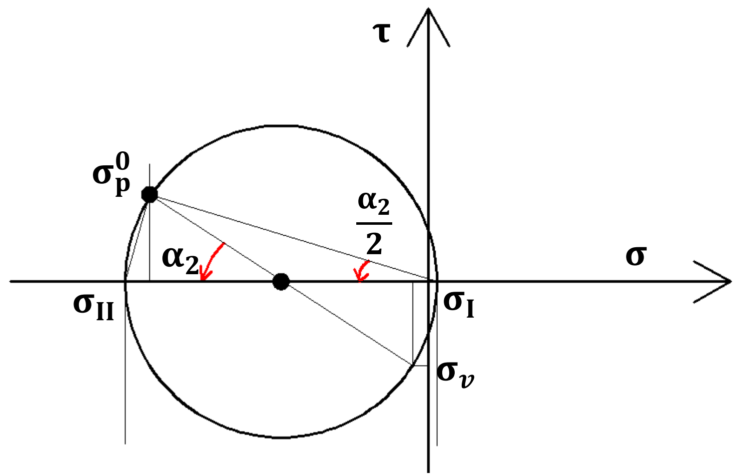
- Transverse prestressing

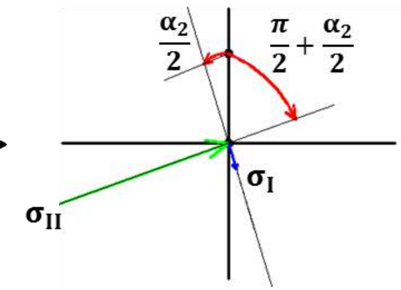
$$\sigma_I = \frac{\sigma_p^0 + \sigma^v}{2} + \frac{1}{2}\sqrt{\left(\sigma_p^0 - \sigma^v\right)^2 + 4\tau^2}$$

$$\sigma_{II} = \frac{\sigma_p^0 + \sigma^v}{2} - \frac{1}{2}\sqrt{\left(\sigma_p^0 - \sigma^v\right)^2 + 4\tau^2}$$

$$\sigma_I = 0 \quad \Rightarrow \quad \sigma_p^0 \sigma^v = \tau^2 \;\; ; \sigma^v = \frac{\tau^2}{\sigma_p^0}$$

$$tg\left(\frac{\alpha_2}{2}\right) = \frac{\tau}{\sigma_{II}} = \frac{\sqrt{\sigma_p^0 \sigma^v}}{2\left(\sigma_p^0 + \sigma^v\right)}$$




- Ultimate limit state, no tension material:

$$V_{Rd,c} = \frac{Ib_w}{S}\sqrt{(f_{ctd})^2 + \sigma_{cp} \cdot f_{ctd}}$$

$$\tau_{max} = \sqrt{(f_{ctd})^2 + \sigma_{cp} \cdot f_{ctd}} \qquad \sigma_I = f_{ctd}$$

# SHEAR LIMIT STATE

## Members without transverse reinforcement

$$V_{Sd}^{*} = V_{Sd} - N\sin\alpha$$

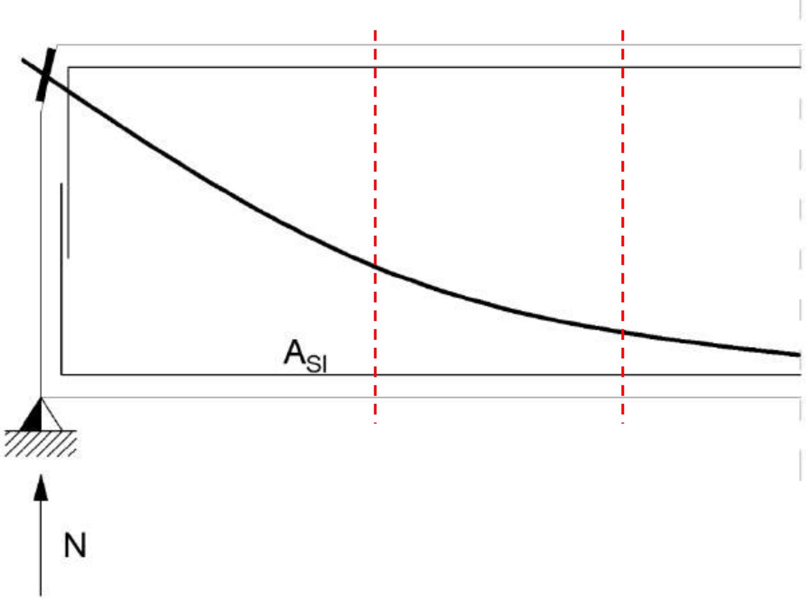
$$V_{Rdc} = \left[\frac{0.18}{\gamma_c} k(100\rho_l f_{ck})^{\frac{1}{3}} + k_1\sigma_{cp}\right] bd$$

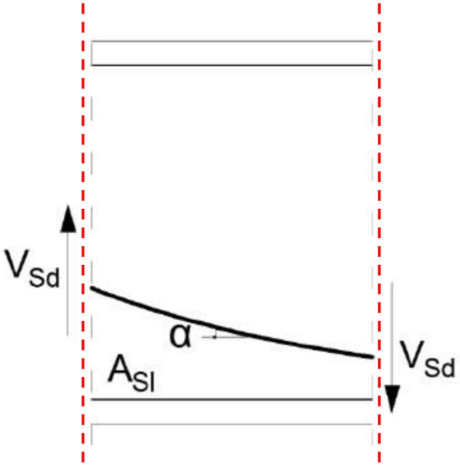
$$k_1 = 1\sqrt{\frac{200}{d}} \leq 2$$

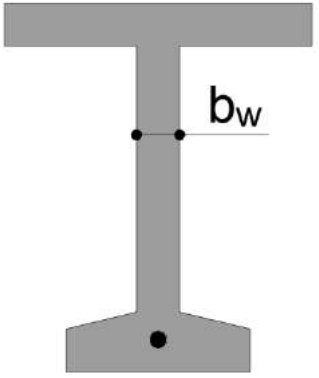
$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0.02$$

$$k_1 = 0.15$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} \leq 0.2f_{cd}$$



### Members with transverse reinforcement

$$t_{Rdc} = \frac{\tau_{Rdc}}{\nu_{fcd}} = \frac{\text{cotg}\theta}{1+\text{cotg}\theta}$$

$$t_{Rd,ss} = \omega_{ss}\text{cotg}\theta$$

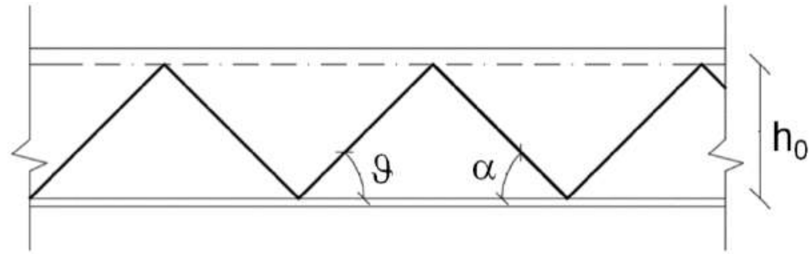
$$t_{Rd,sl} = \frac{2.22}{\delta}\omega_{sl}\text{tg}\theta$$

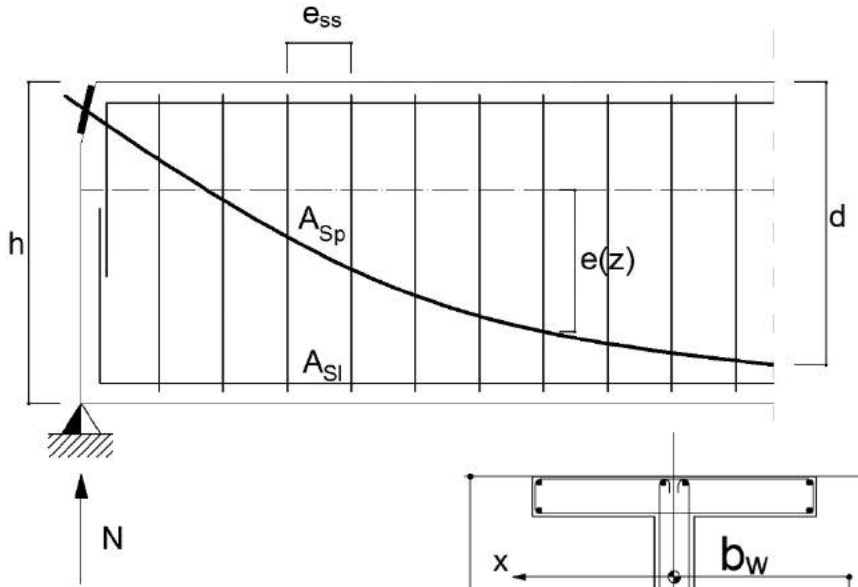
$$t_{Ed} = \frac{\left(V_{Ed} - N_{Pd}e'(z)\right)}{0.9b_w d}$$

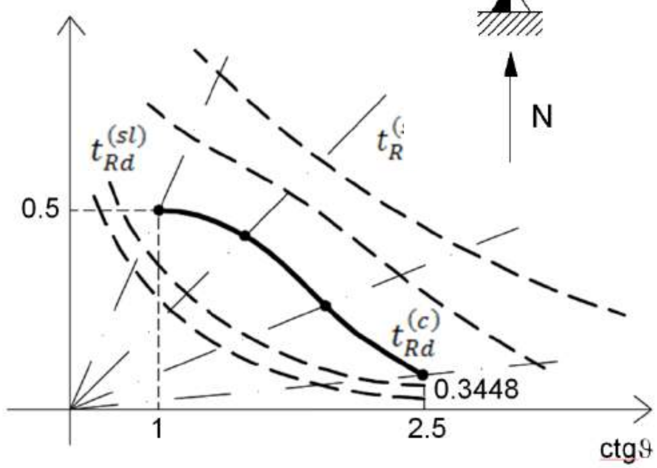
$$\omega_{ss} = \frac{A_{ss}f_{yd}}{b_w e_{ss}\nu f_{cd}}$$

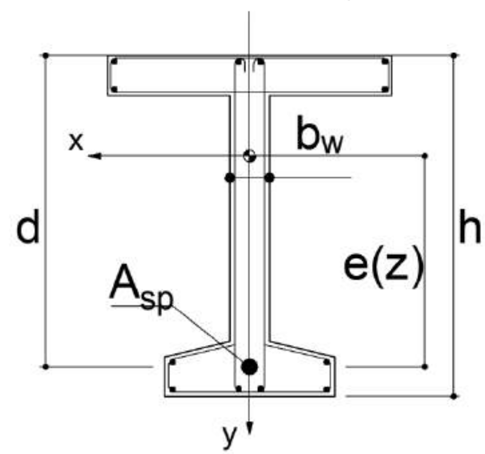
$$\omega_{sl} = \frac{A_{sl}f_{yd}}{b_w h\nu f_{cd}}$$

$$\delta = \frac{d}{h}$$



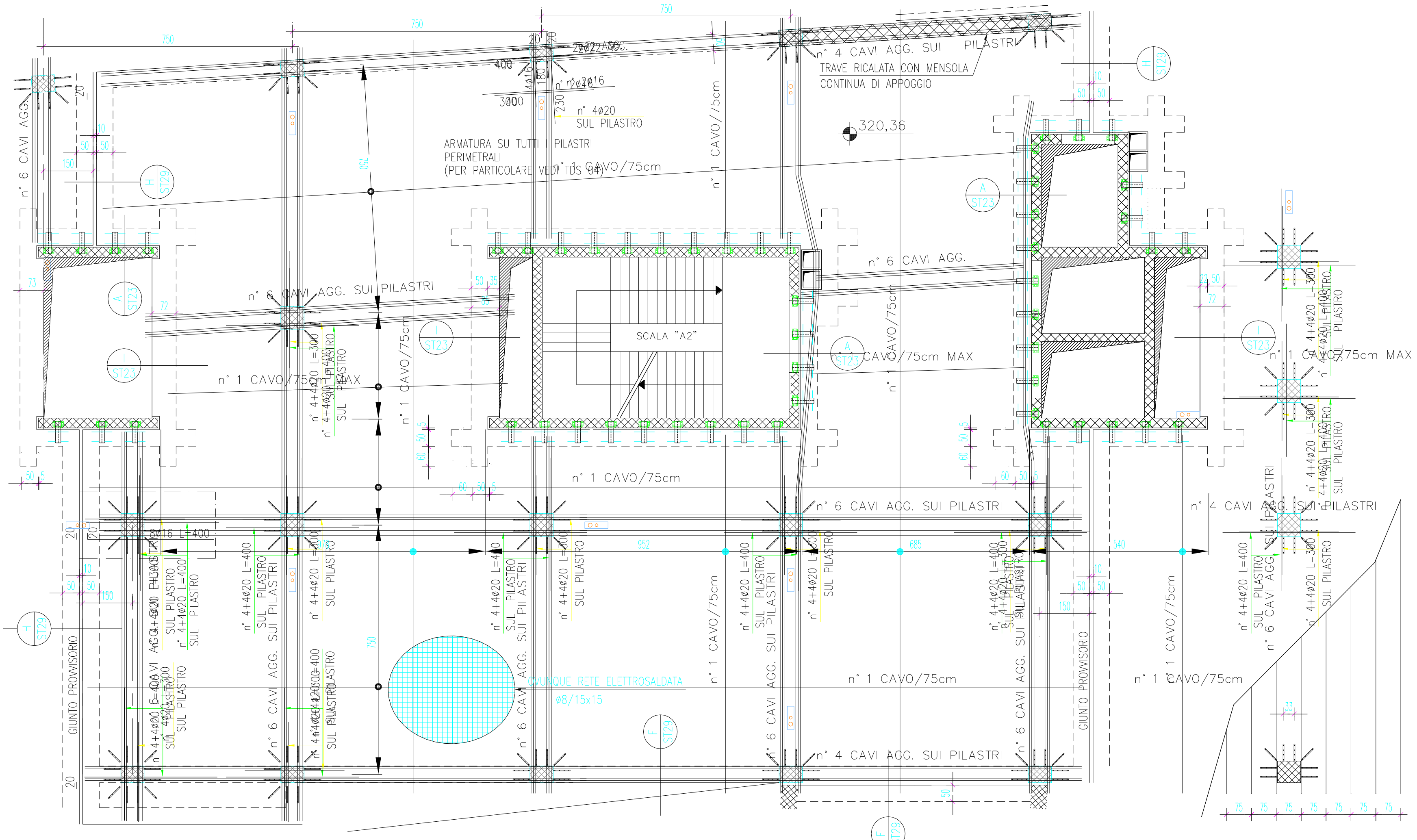



# CASE STUDIES

# CASE STUDIES


# CASE STUDIES – CONSTRUCTIONAL DETAILS



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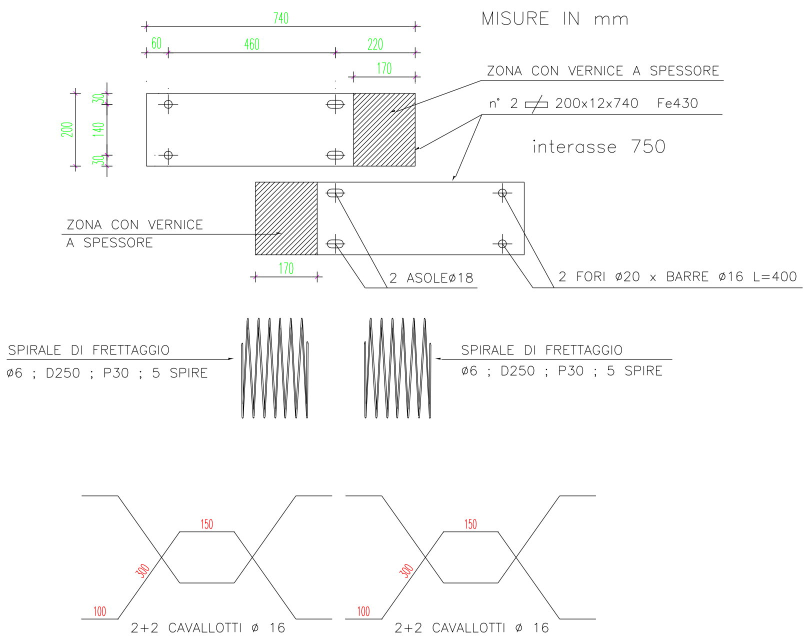
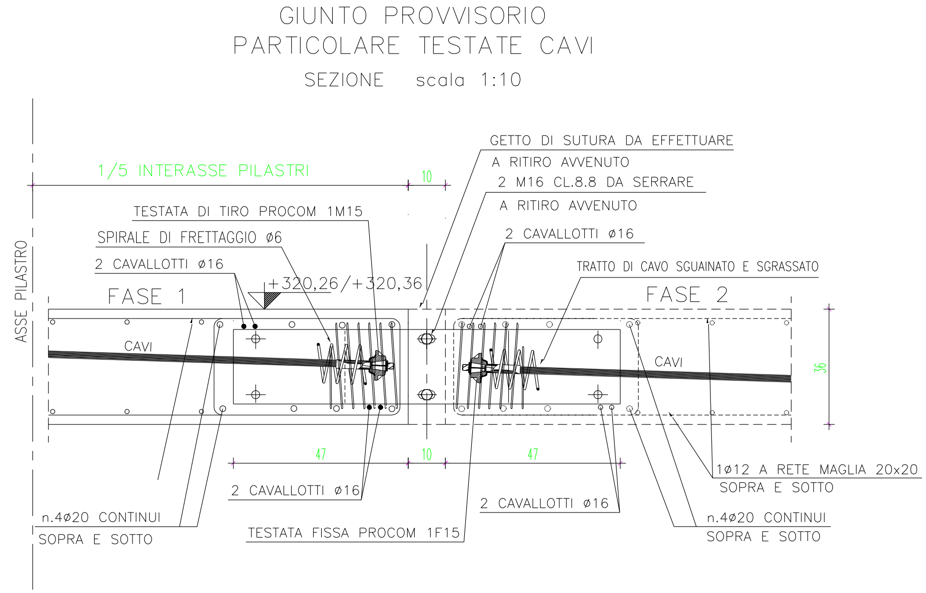
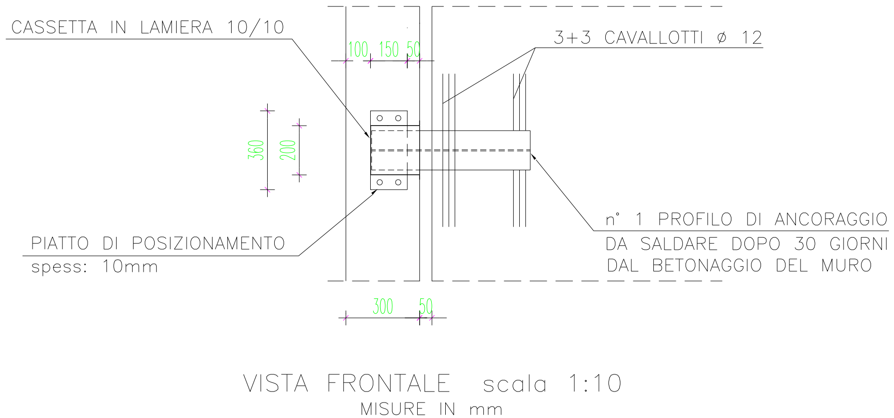
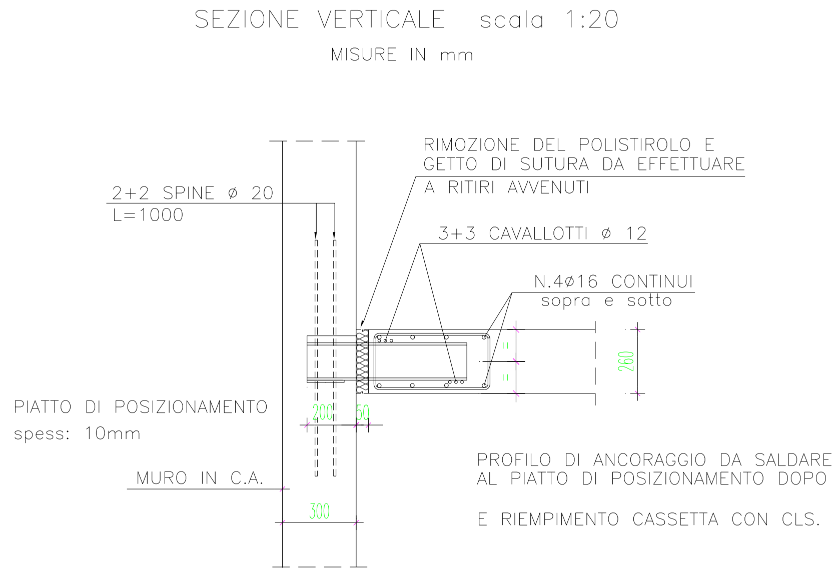
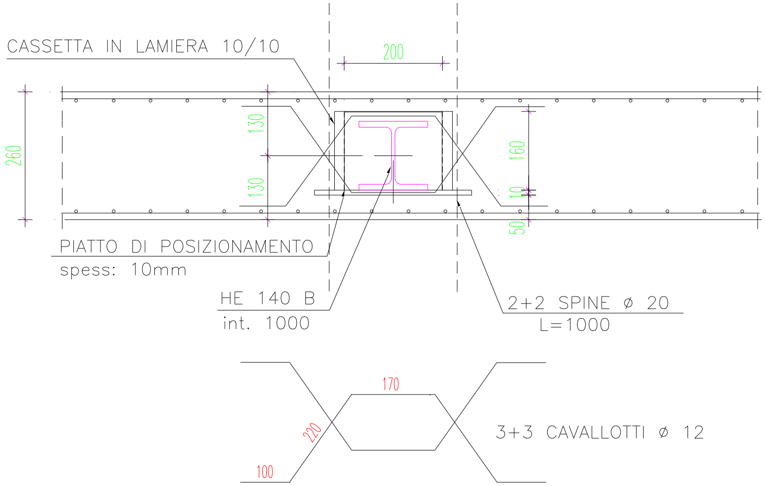


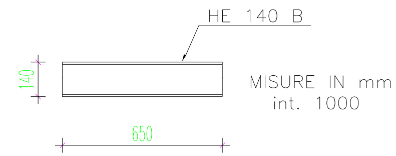



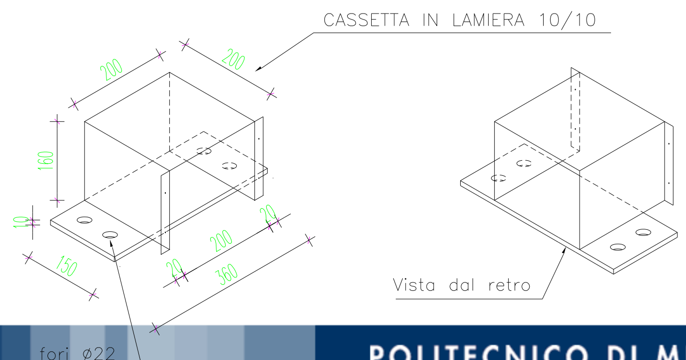
# CASE STUDIES – CONSTRUCTIONAL DETAILS



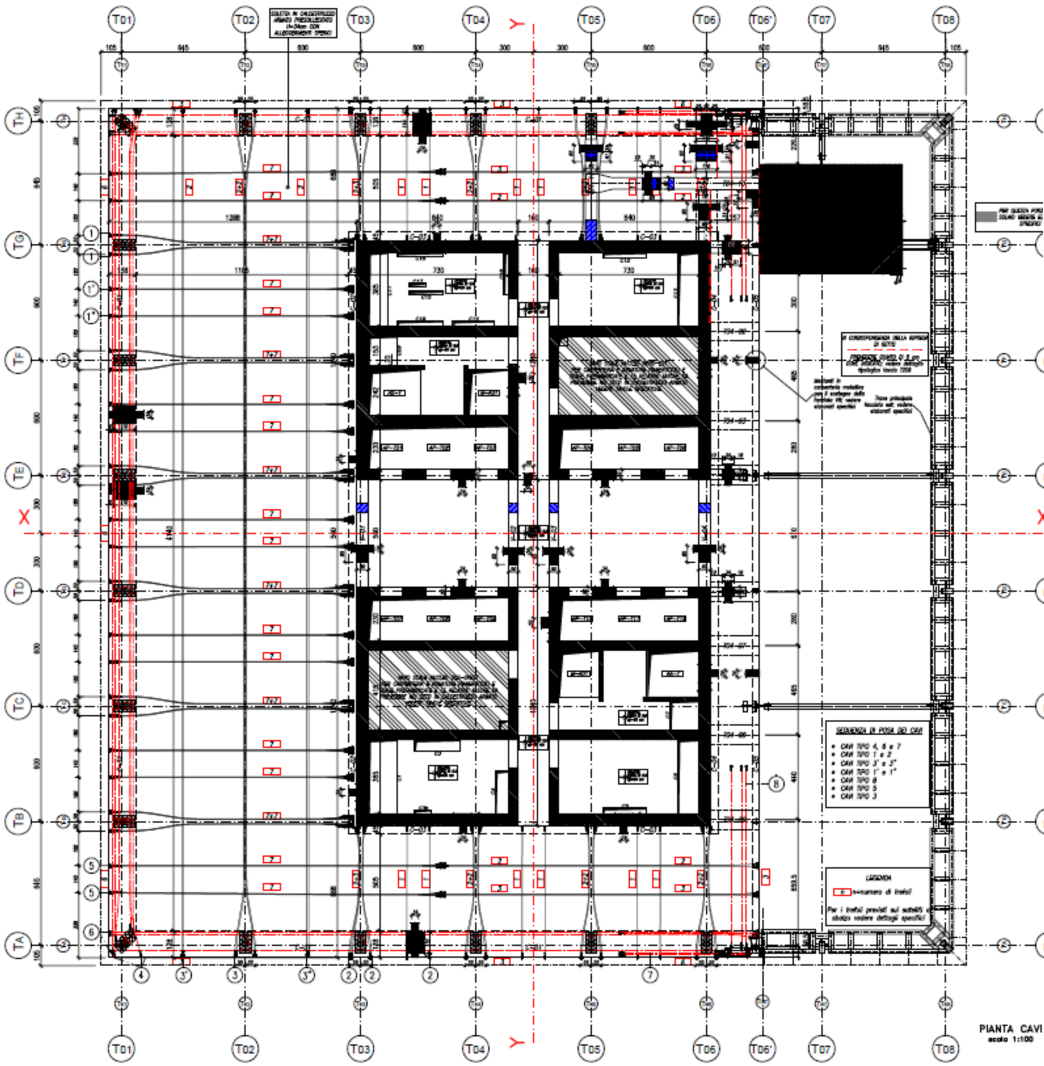



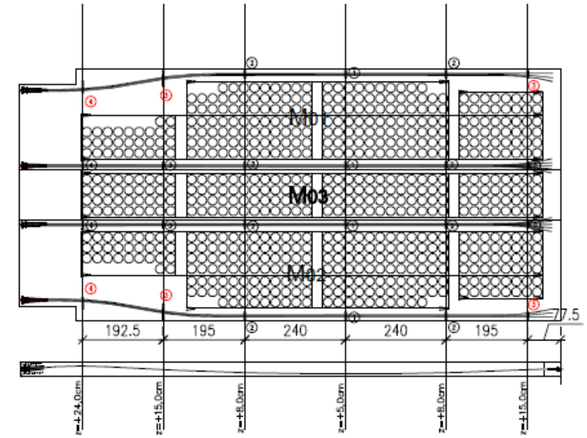
# CASE STUDIES - REGIONE PIEMONTE HEADQUARTERS

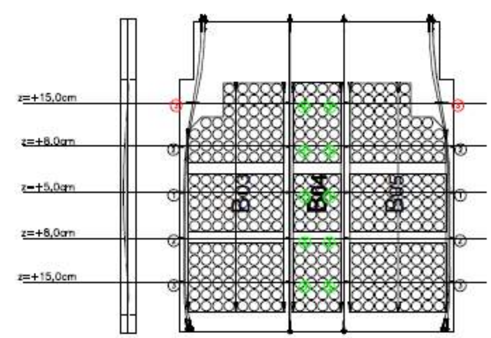
# CASE STUDIES - REGIONE PIEMONTE HEADQUARTERS

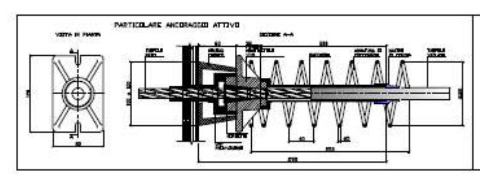
# CASE STUDIES - REGIONE PIEMONTE HEADQUARTERS

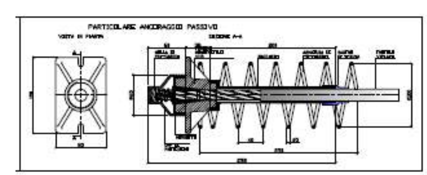
## Standard Floor: lightened slab, h = 34 cm, prestressed with unbonded strands



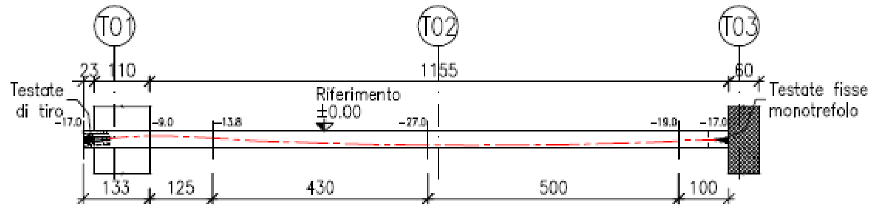



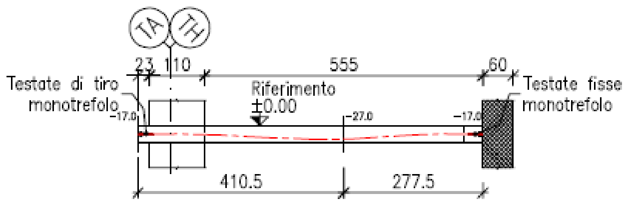
## Standard Floor: lightened slab, h = 34 cm, prestressed with unbonded strands

| |
|---|
| $\sigma = 0.75 \times f_{ptk} = 1395$ MPa |
| $R_{ck,j} \geq 35$ MPa |
| $E_{cm,j} \geq 20$ GPa |

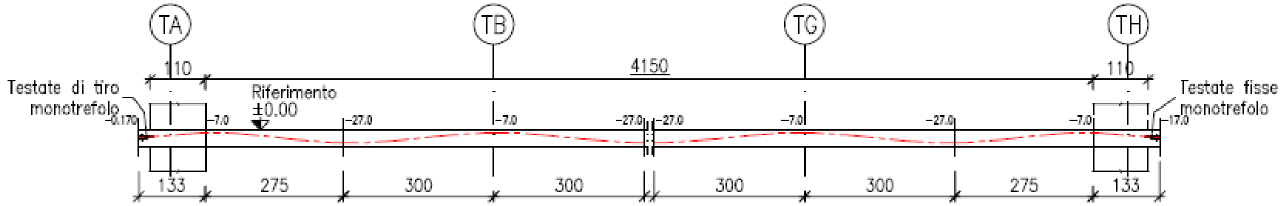
CAVI TIPO 1, 1' e 1" –CAMPATA DA 12m–
TIPOLOGICO TRACCIATO ALTIMETRICO
scala 1:100




CAVI TIPO 2 –CAMPATA DA 6m–
TIPOLOGICO TRACCIATO ALTIMETRICO
scala 1:100




CAVI TIPO 3, 3', 3" e 4
TIPOLOGICO TRACCIATO ALTIMETRICO
scala 1:100



# CASE STUDIES - REGIONE PIEMONTE HEADQUARTERS

## Structural Analysis of the Standard Floor

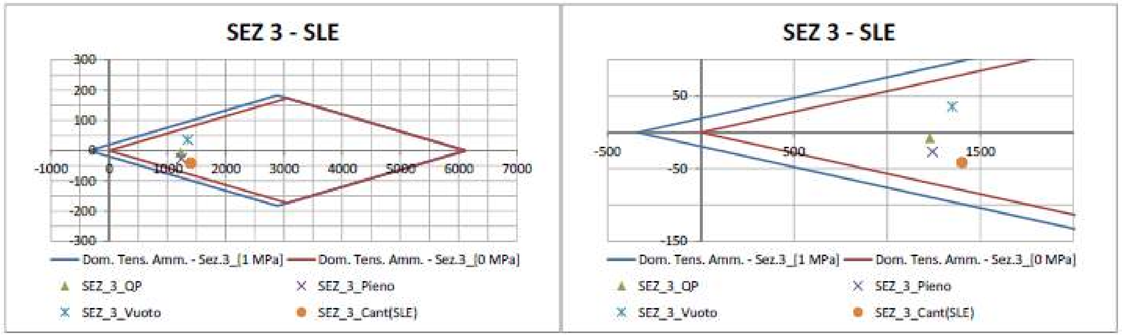
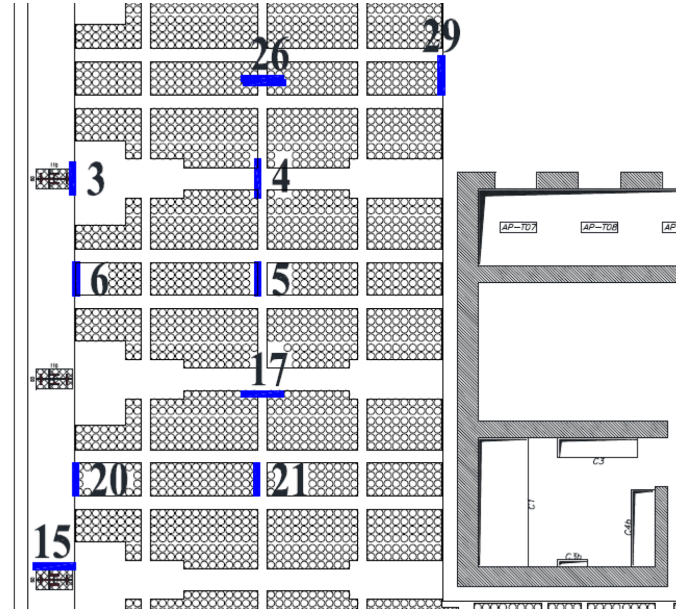
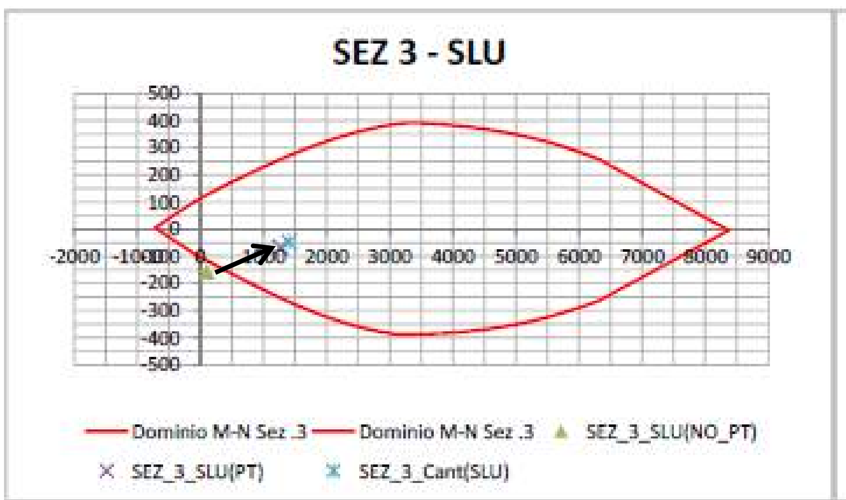
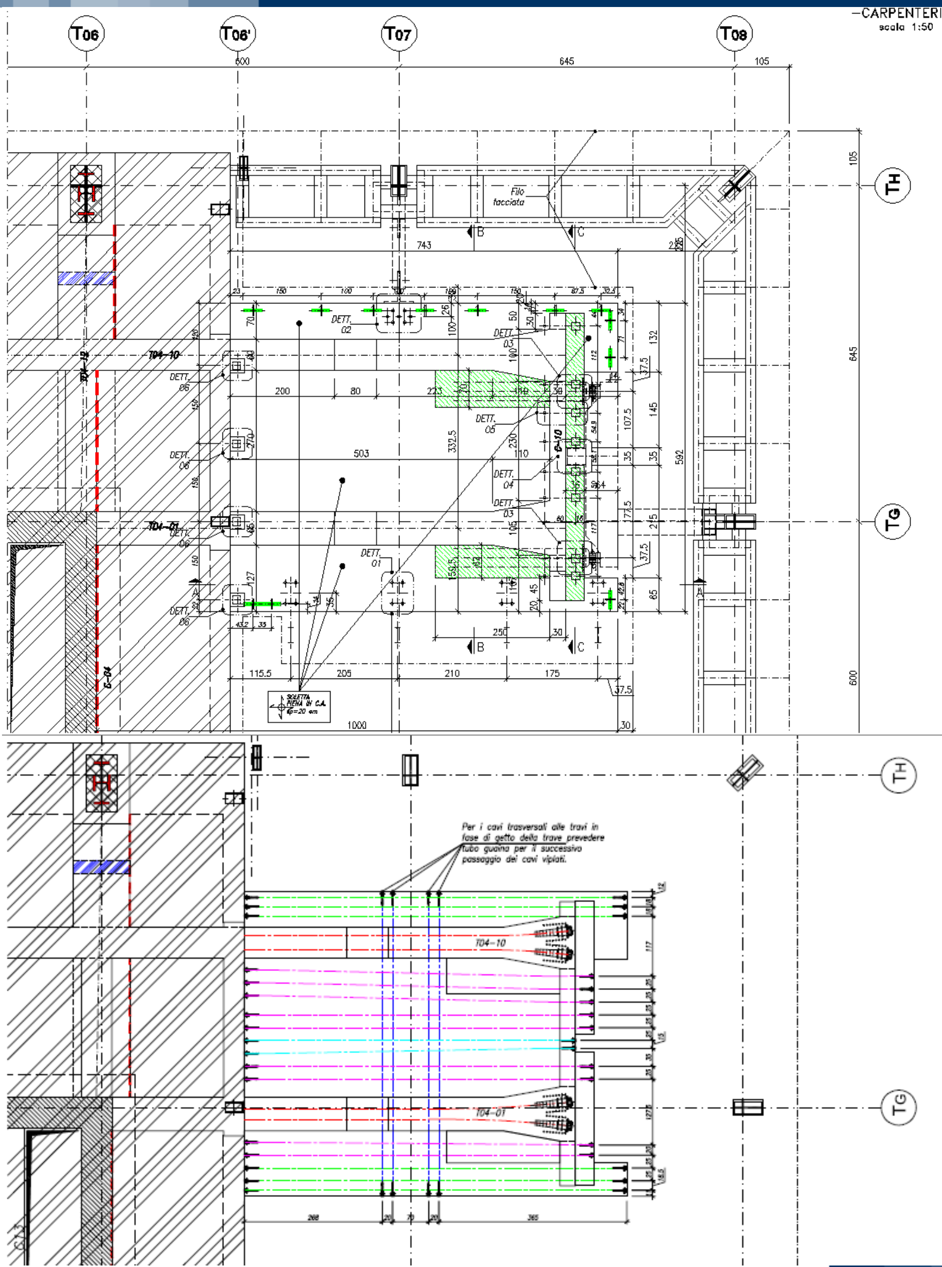



Figura 44 - Verifica dello stato tensionale della sezione 3 a Vuoto, a Pieno, in combinazione rara e di cantiere



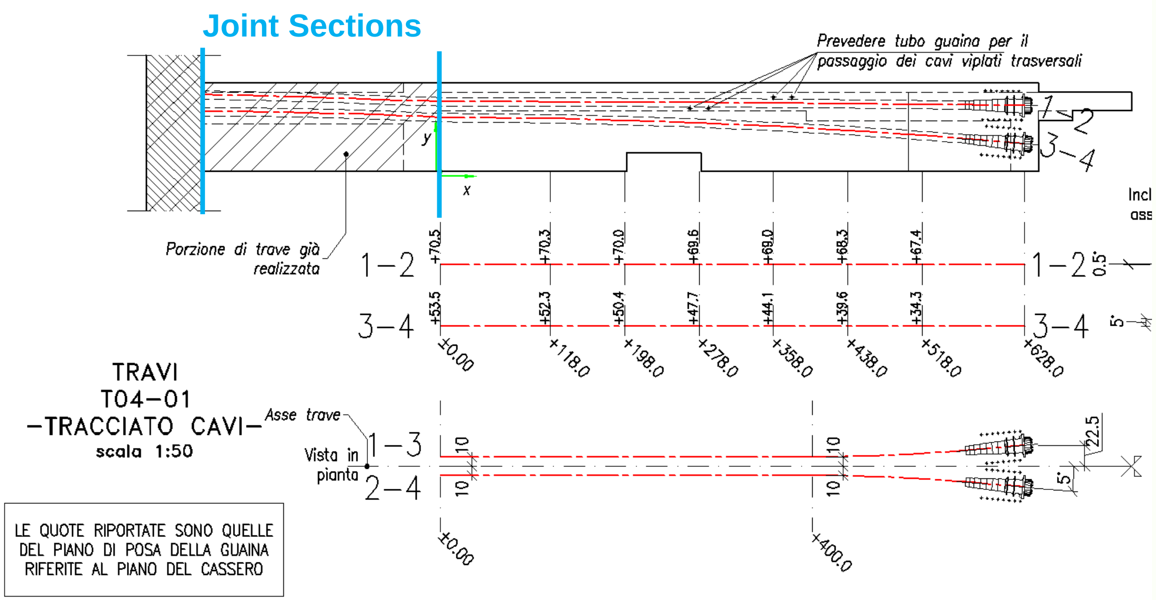

# CASE STUDIES - REGIONE PIEMONTE HEADQUARTERS



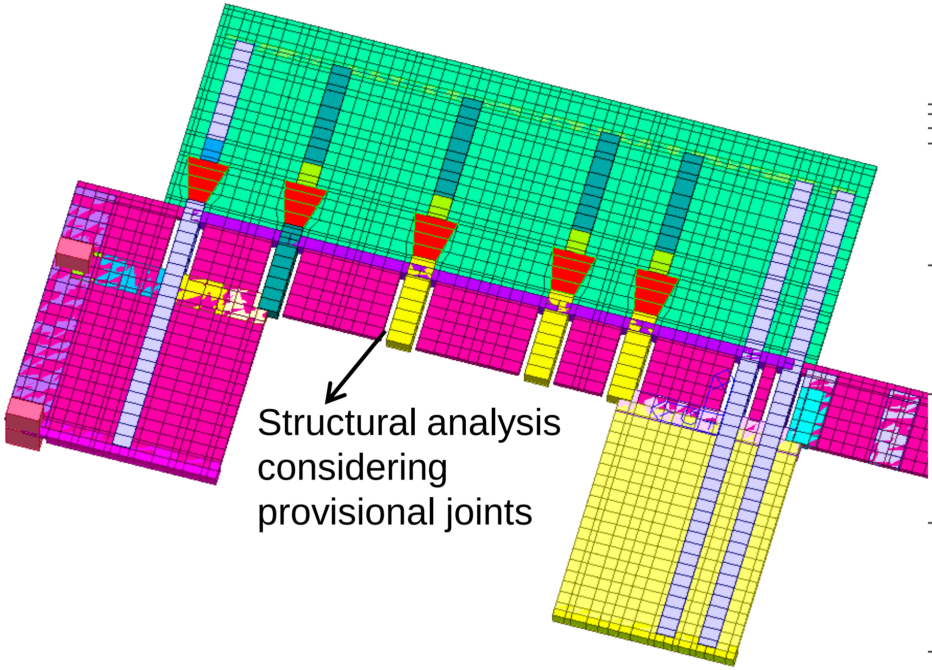

## East Façade "Jump Deck" Floors

# CASE STUDIES - REGIONE PIEMONTE HEADQUARTERS

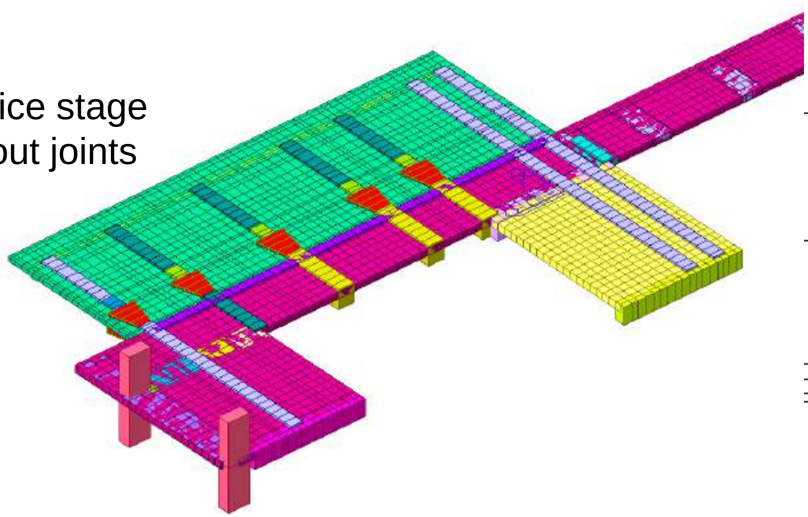
## The cantilevered prestressed concrete floor structures (“Jump-deck floors”)




# CASE STUDIES - REGIONE PIEMONTE HEADQUARTERS: Structural Analysis for the 'Jump deck Floor'

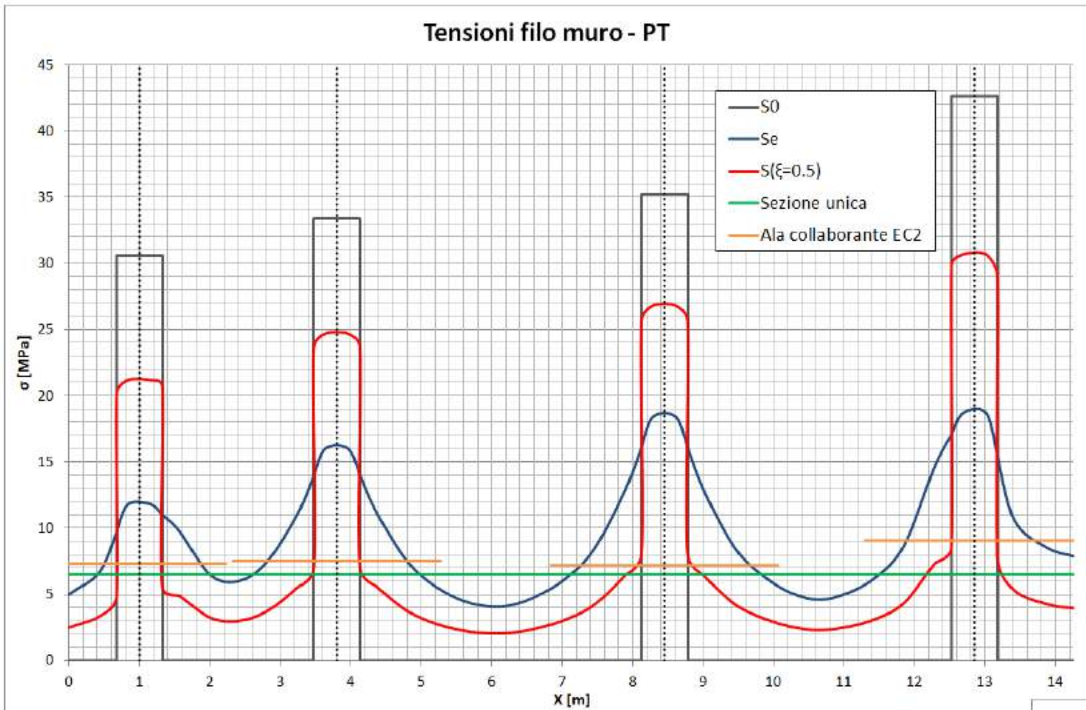


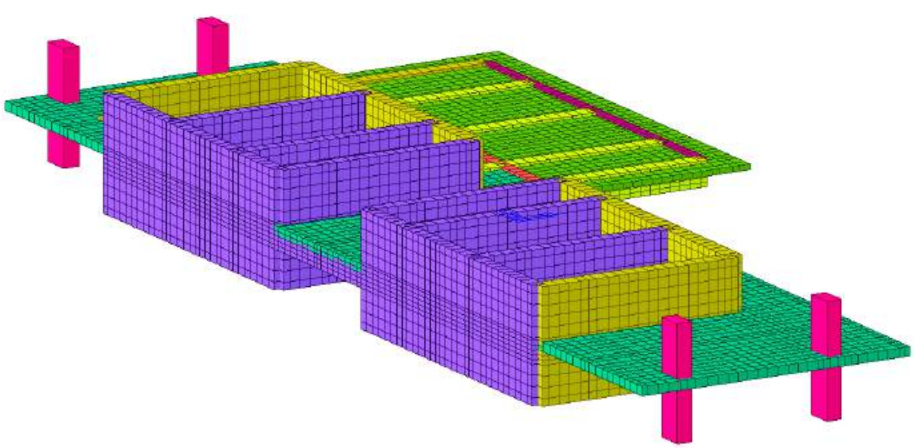
Structural analysis considering provisional joints

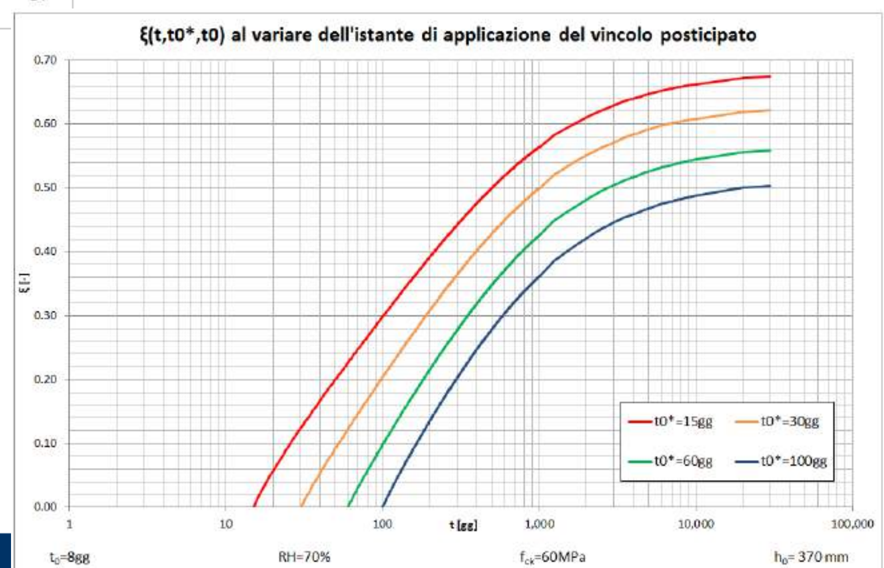


Service stage without joints

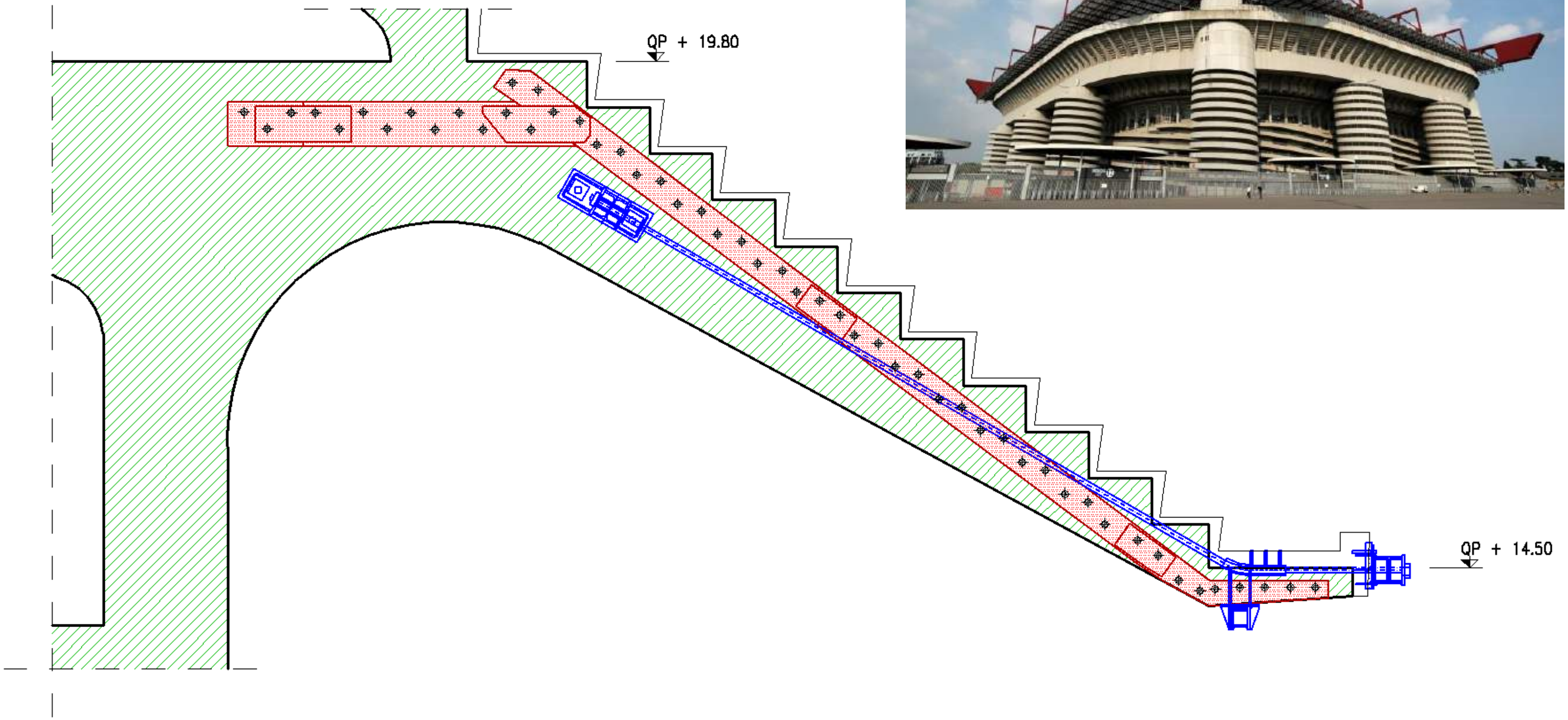
# CASE STUDIES - REGIONE PIEMONTE HEADQUARTERS



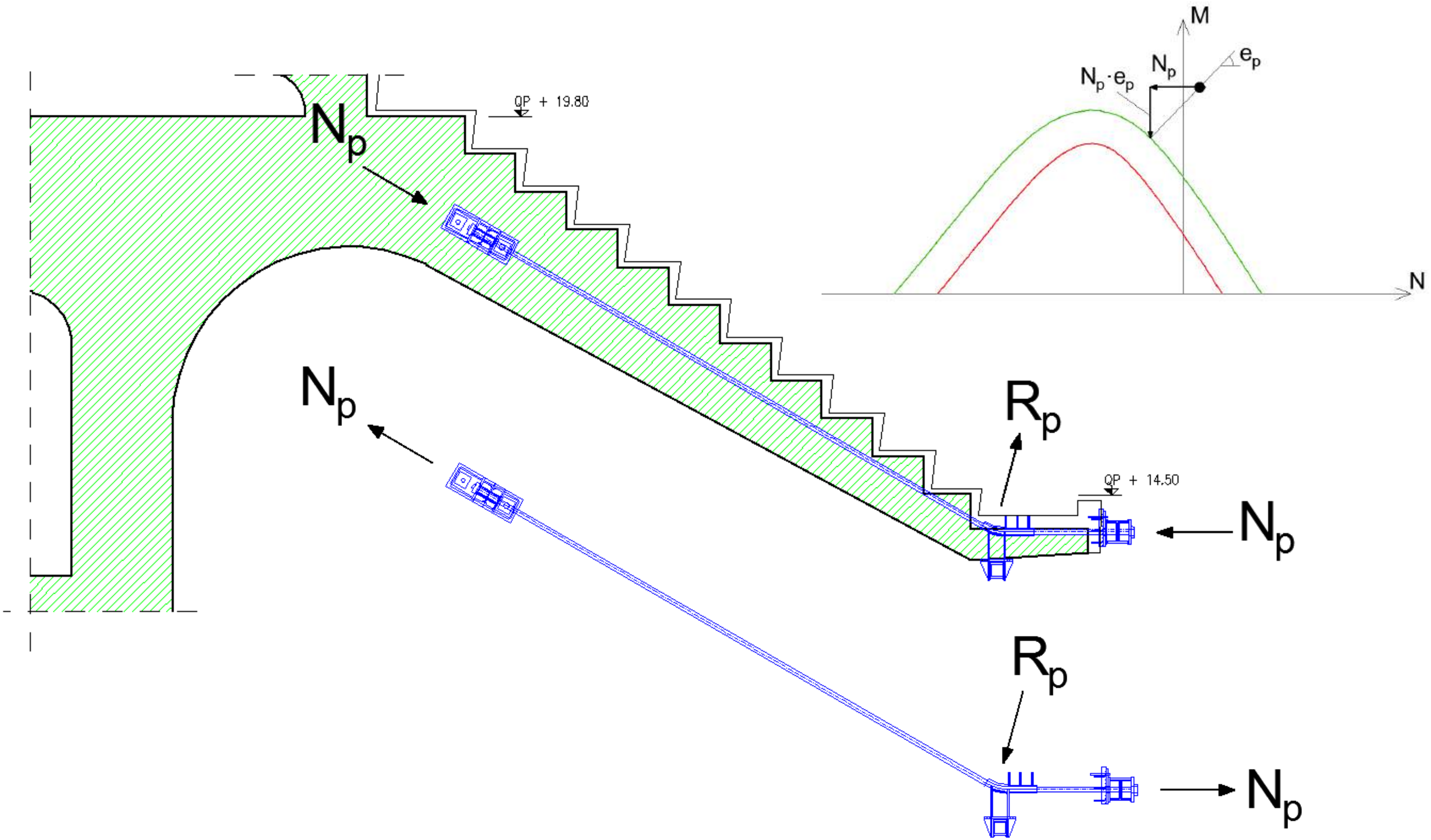





# CASE STUDIES – MEAZZA STADIUM IN MILAN

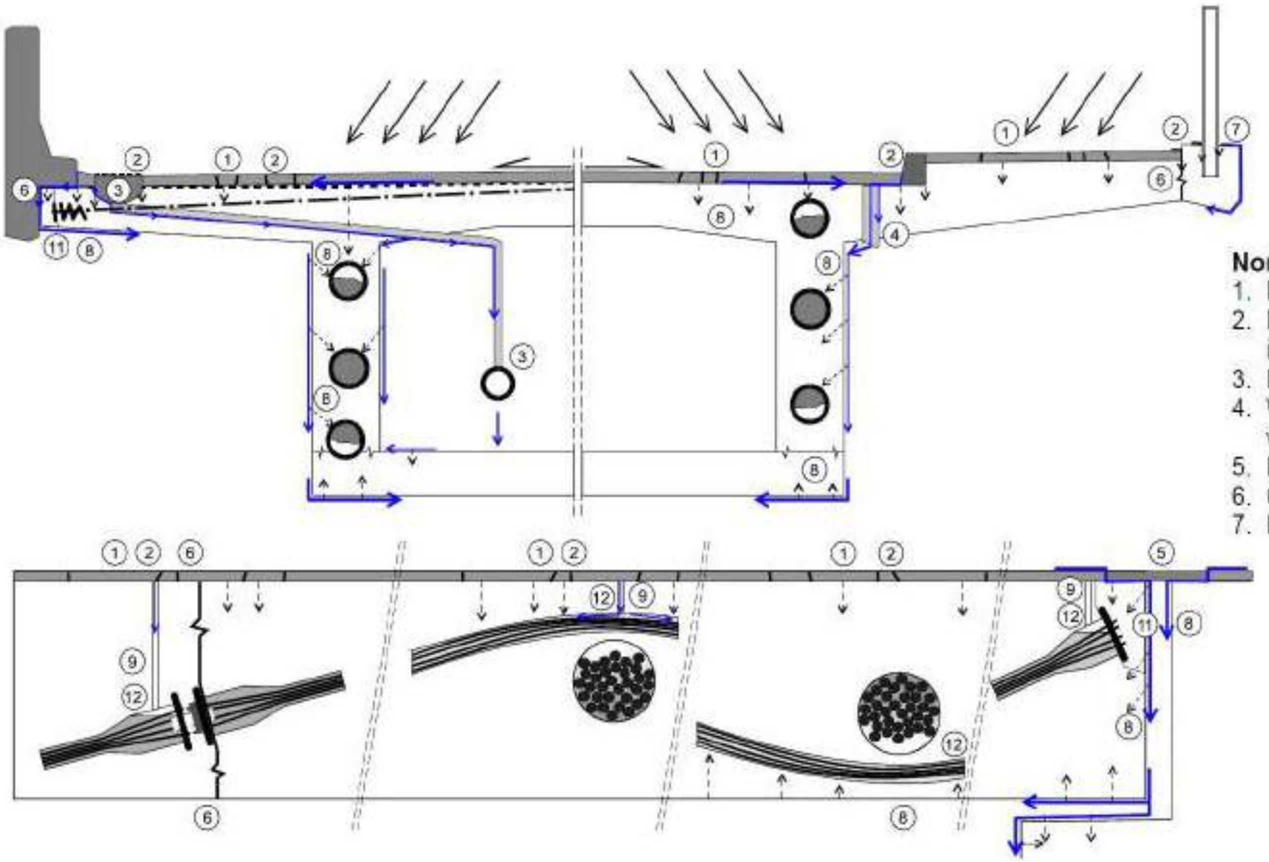


# CASE STUDIES – MEAZZA STADIUM IN MILAN



## STRUCTURAL AND NON-STRUCTURAL CAUSES OF DEGRADATION

Possibili percorsi di penetrazione dell'acqua



**Non-structural elements:**

1. Defective wearing course (e.g. cracks)
2. Missing or defective waterproofing membrane incl. edge areas
3. Defective drainage intakes and pipes
4. Wrongly placed outlets for the drainage of wearing course and waterproofing
5. Leaking expansion joints
6. Cracked and leaking construction or element joints
7. Inserts (e.g. for electricity)

**Corrosion protection system :**

8. Defective concrete cover
9. Partly or fully open grouting in- and outlets (vents)
10. Leaking, damaged metallic ducts mechanically or by corrosion
11. Cracked and porous pocket concrete
12. Grout voids at tendon high points

Note: In precast segmental construction the dry packing of lifting holes, and stressing pockets in segment faces need to be checked.

# DEGRADING AND FAILURE OF P.C STRUCTURES

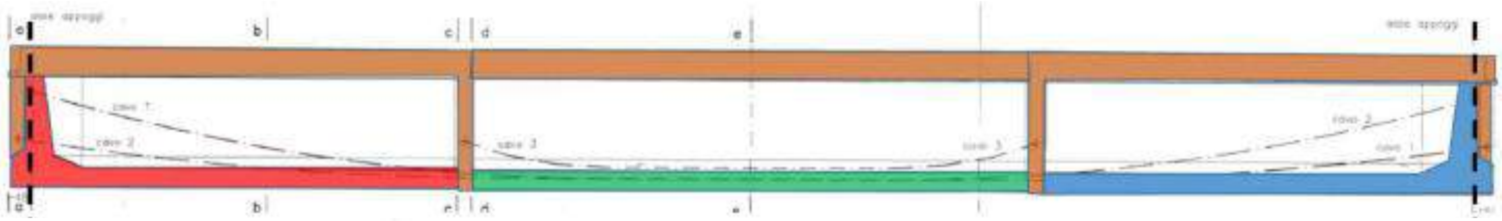
Situazione Ante Crollo

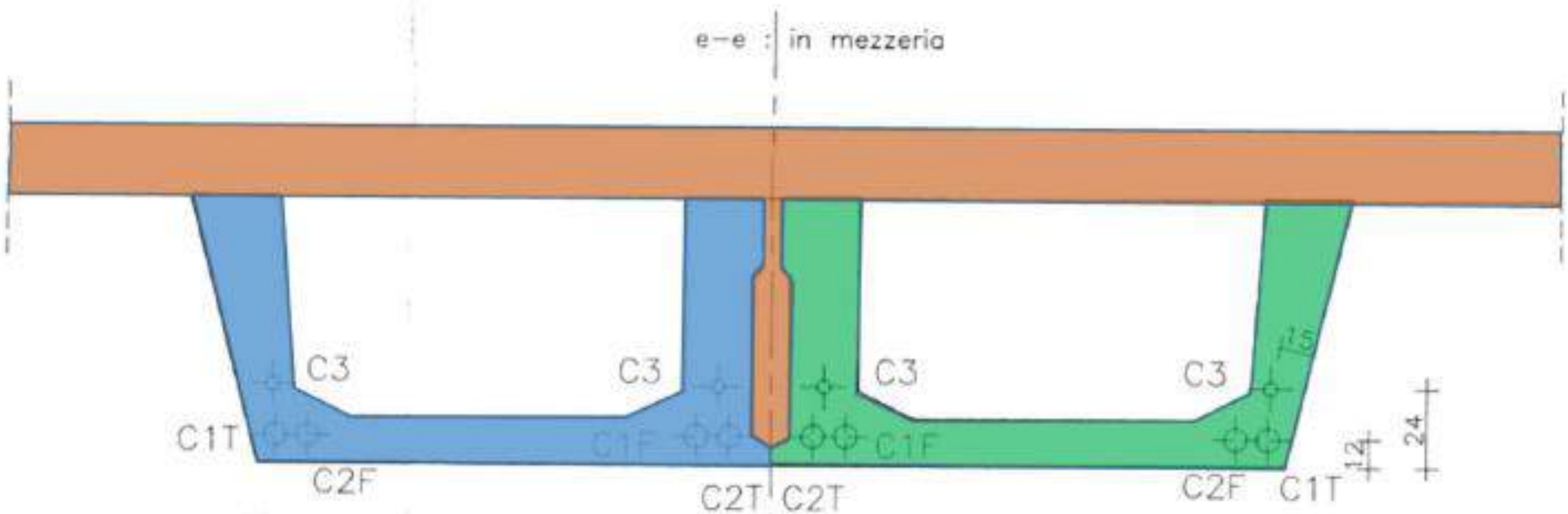
Situazione Post Crollo (Attuale)

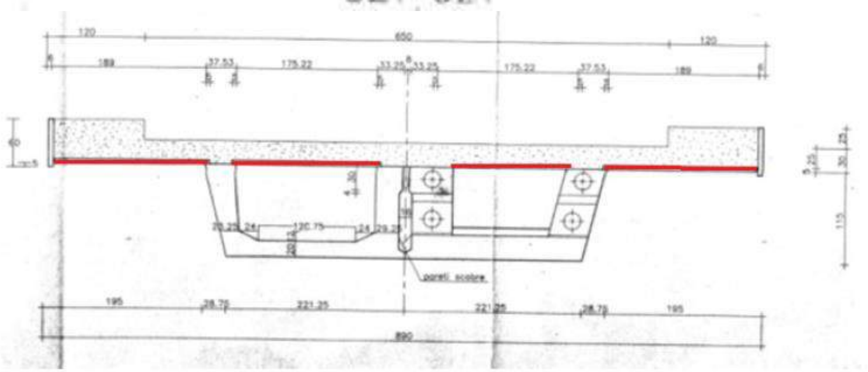
# DEGRADING AND FAILURE OF P.C STRUCTURES



# DEGRADING AND FAILURE OF P.C STRUCTURES

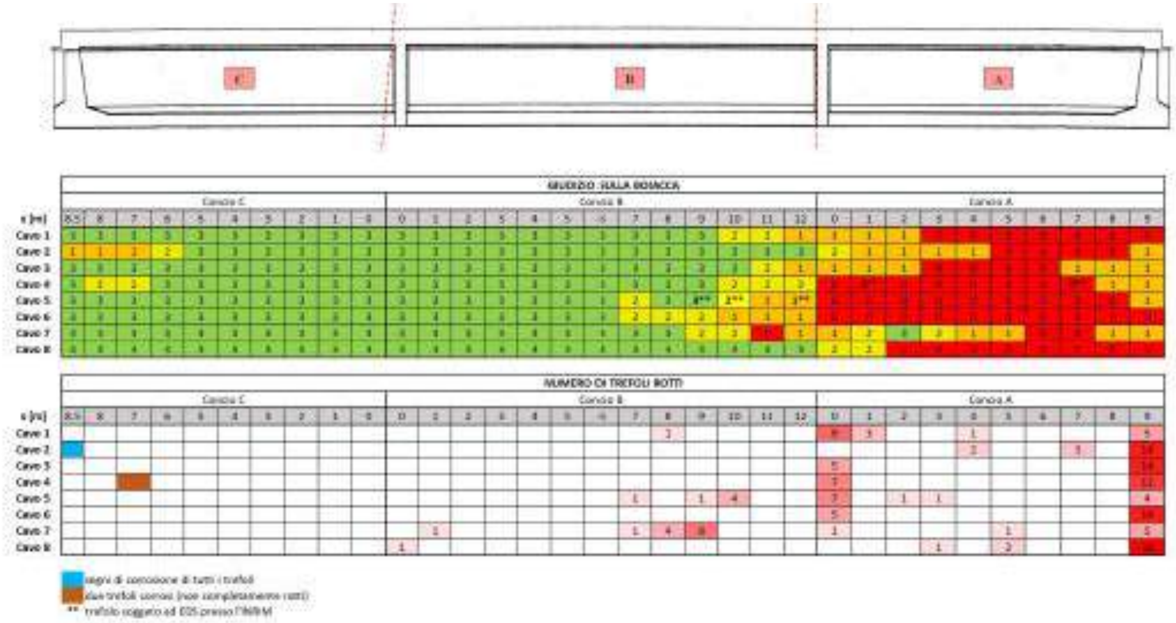




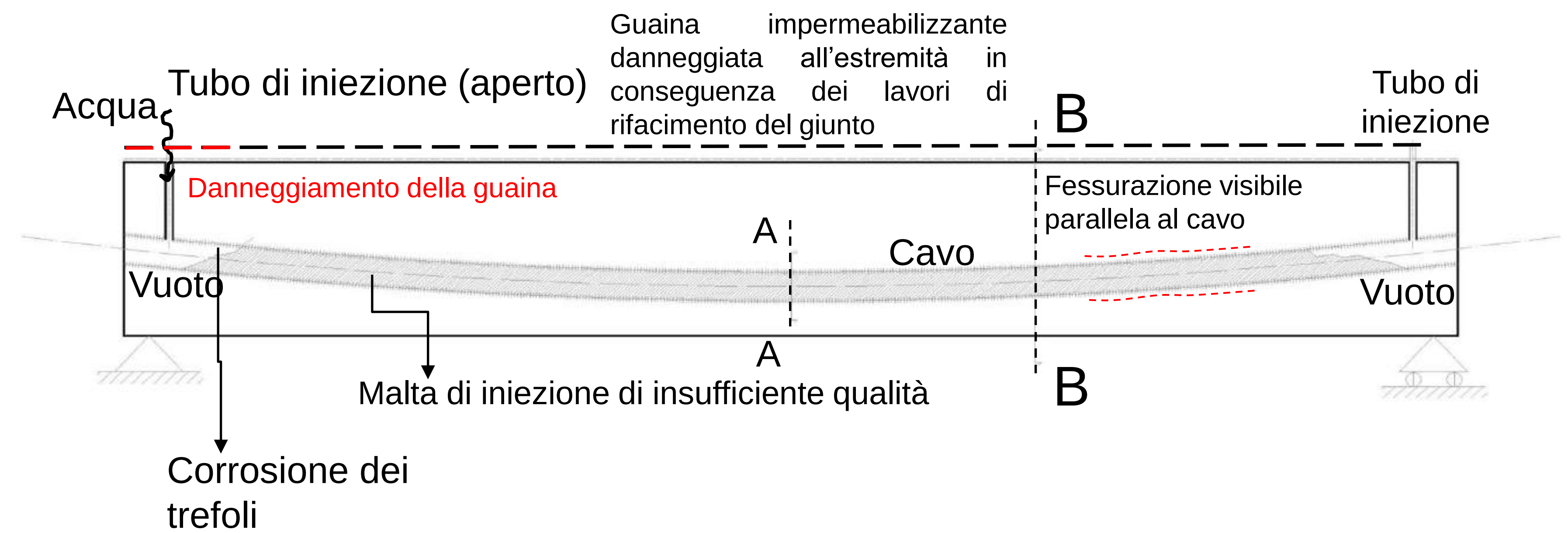



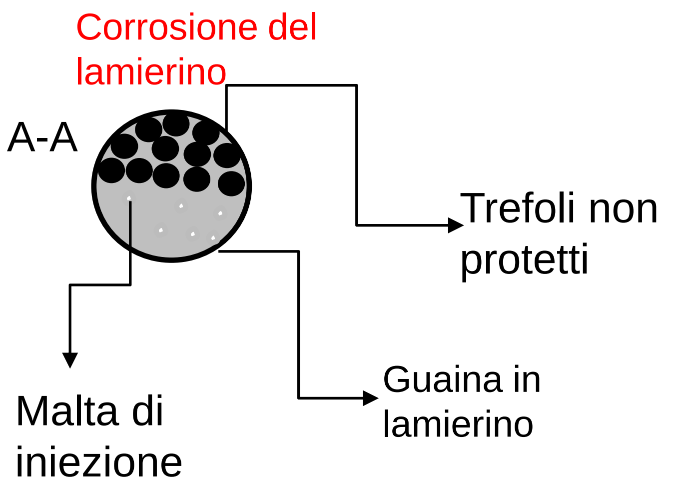
# DEGRADING AND FAILURE OF P.C STRUCTURES

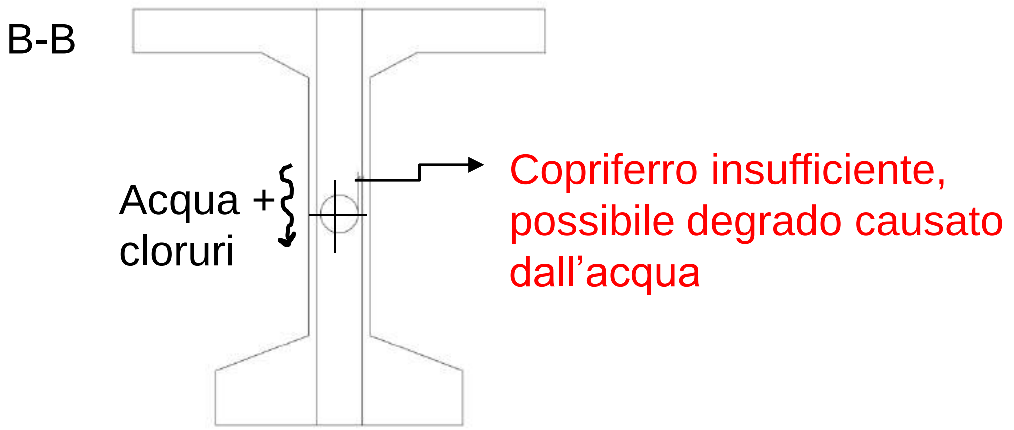
# DEGRADING AND FAILURE OF P.C STRUCTURES



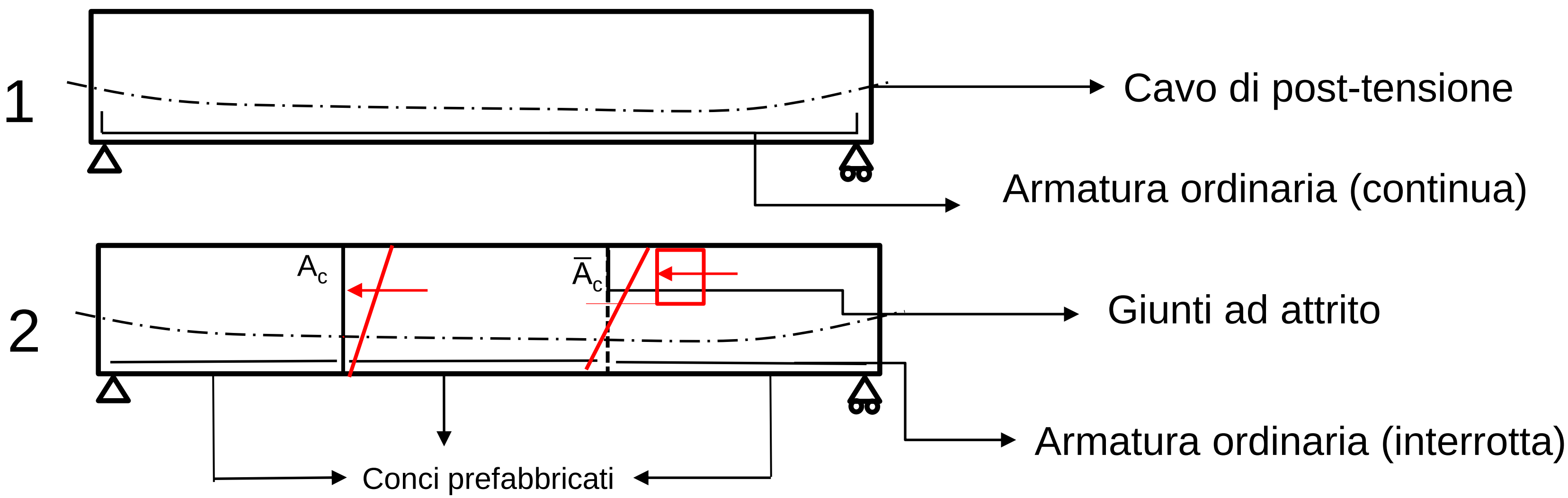

## LE CAUSE DEL DANNEGGIAMENTO DEI TREFOLI



# DEGRADING AND FAILURE OF P.C STRUCTURES

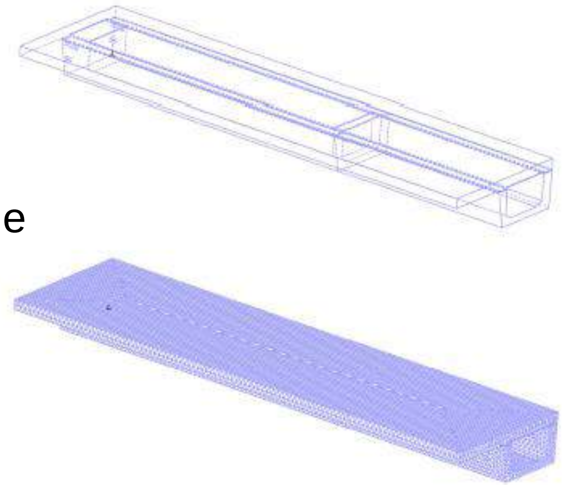



Le due configurazioni strutturali sono marcatamente differenti in termini di robustezza.

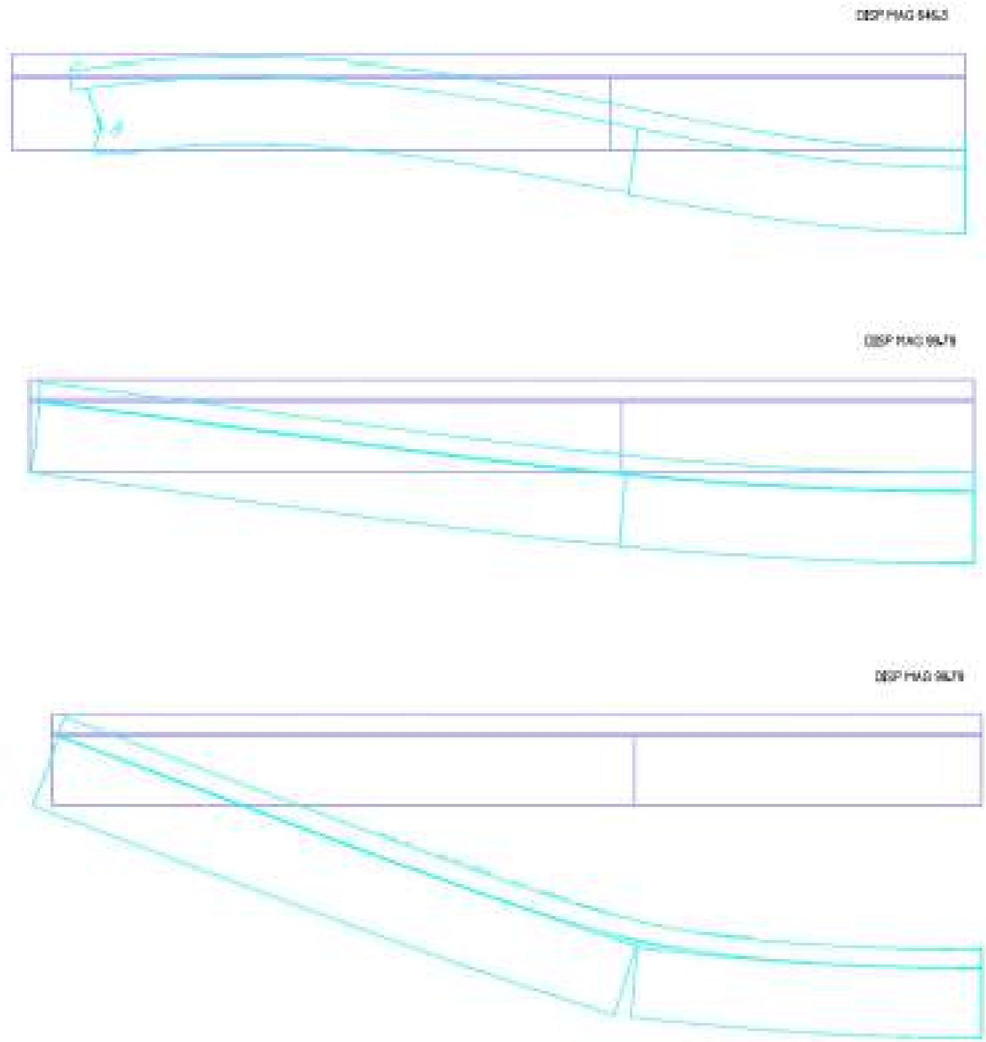
| | Armature sane | | Armature di presollecitazione danneggiate | |
|---|---|---|---|---|
| 1 | $M_{Rd} = M_{Rd,p} + M_{Rd,s}$<br>$V_{Rd} = f_{yd} A_{ss} \frac{0.9d}{e_{ss}} ctg\theta$ | 1 | $M_{Rd} = (M_{Rd,p} - \Delta M_{Rd,p}) + M_{Rd,s}$<br>$V_{Rd} = f_{yd} A_{ss} \frac{0.9d}{e_{ss}} ctg\theta$<br>$M_{Rd} \geq M_{Ek}$ | Assenza di collasso, possibili fessurazioni |
| 2 | $M_{Rd} = M_{Rd,p}$<br>$V_{Rd} = \mu f_{cd} A_c (N_{Rd,p})$ | 2 | $M_{Rd} = (M_{Rd,p} - \Delta M_{Rd,p}) \leq M_{Ek}$<br>$V_{Rd} = \mu f_{cd} \bar{A}_c (N_{Rd,p} - \Delta N_{Rd,p}) \leq V_{Ek}$ | Collasso per flessione o per taglio |

# DEGRADING AND FAILURE OF P.C STRUCTURES

Modello strutturale



Evoluzione del meccanismo di collasso




Spostamenti verticali in funzione della riduzione della presollecitazione


Apertura delle fessure in funzione della riduzione della presollecitazione

# CASE STUDIES: HIGHWAY BRIDGES

***Adda Viaduct:***

- 1260 m long, 20 spans;
- Lenght of the span: 60m, 75m, 90m;
- Depth: 5.40m-3.20m.

# CASE STUDIES: HIGHWAY BRIDGES

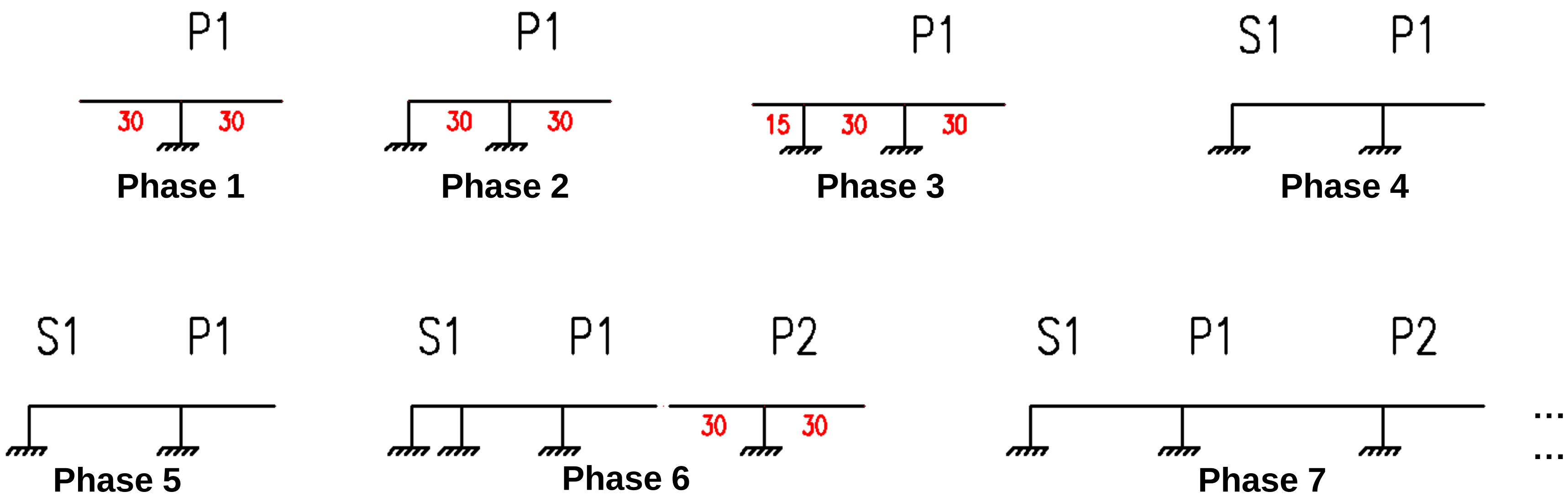
***Serio Viaduct:***

- 930 m long, 16 spans;
- Lenght of the span: 60m;
- Depth: 3.20m.

## *Oglio Viaduct:*

- 6900 m long, 11 spans;
- Lenght of the span: 60m, 75m, 90m.
- Depth: 5.40m-3.20m.

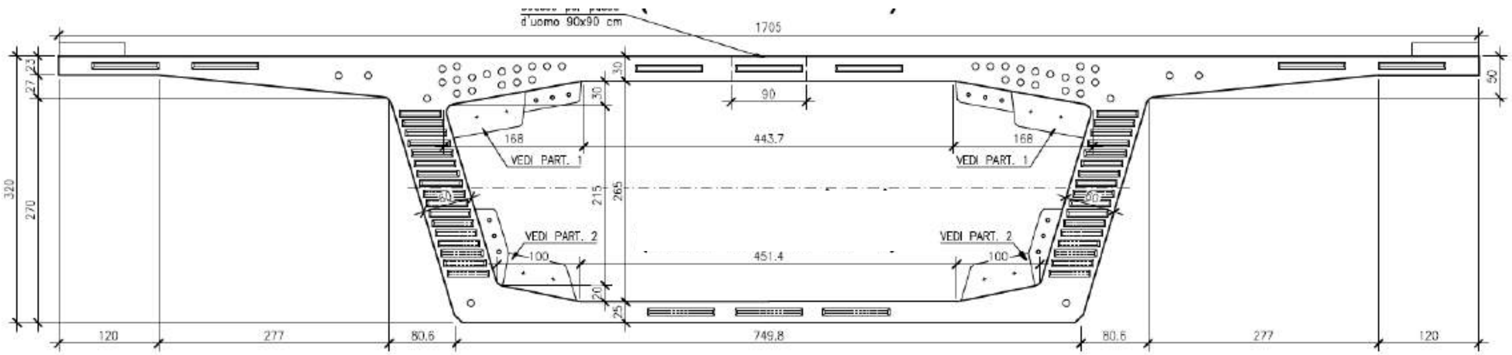
# CASE STUDIES: HIGHWAY BRIDGES




The installation of two temporary supports at the end piers is required: the provisional supports are then removed after the end of the construction

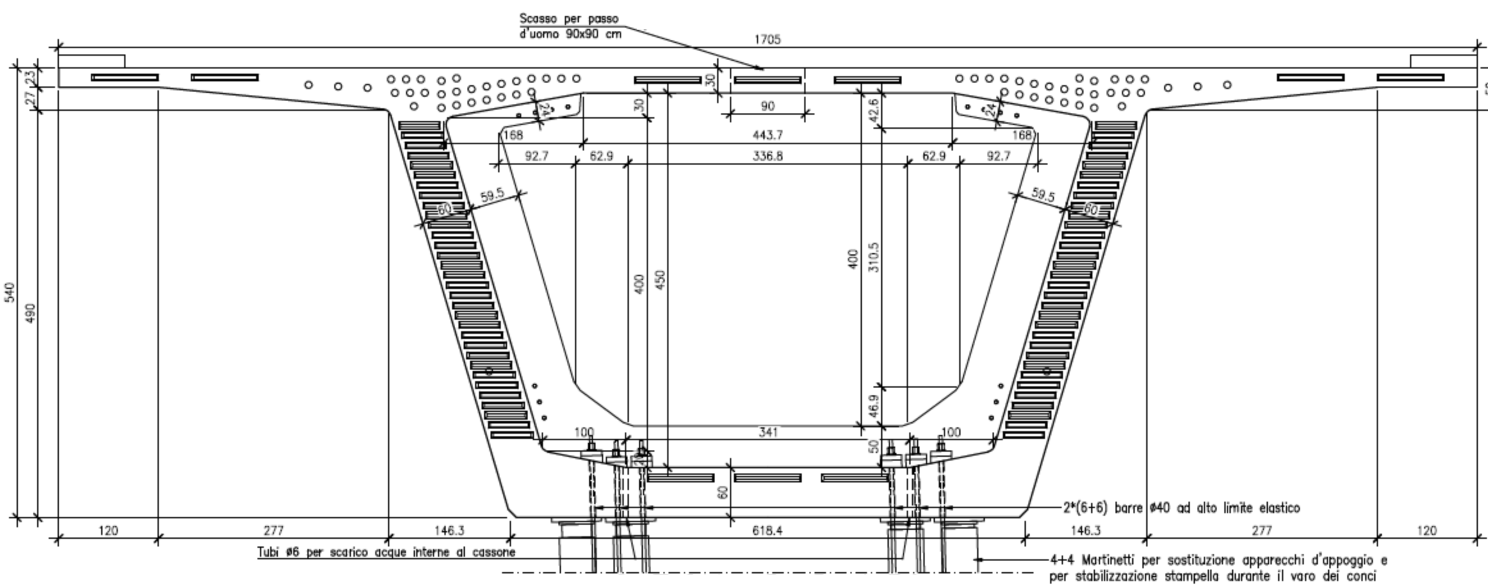
# CASE STUDIES: HIGHWAY BRIDGES

# CASE STUDIES: HIGHWAY BRIDGES




Depth: 3.20m

Depth: 5.40m



# CASE STUDIES: HIGHWAY BRIDGES

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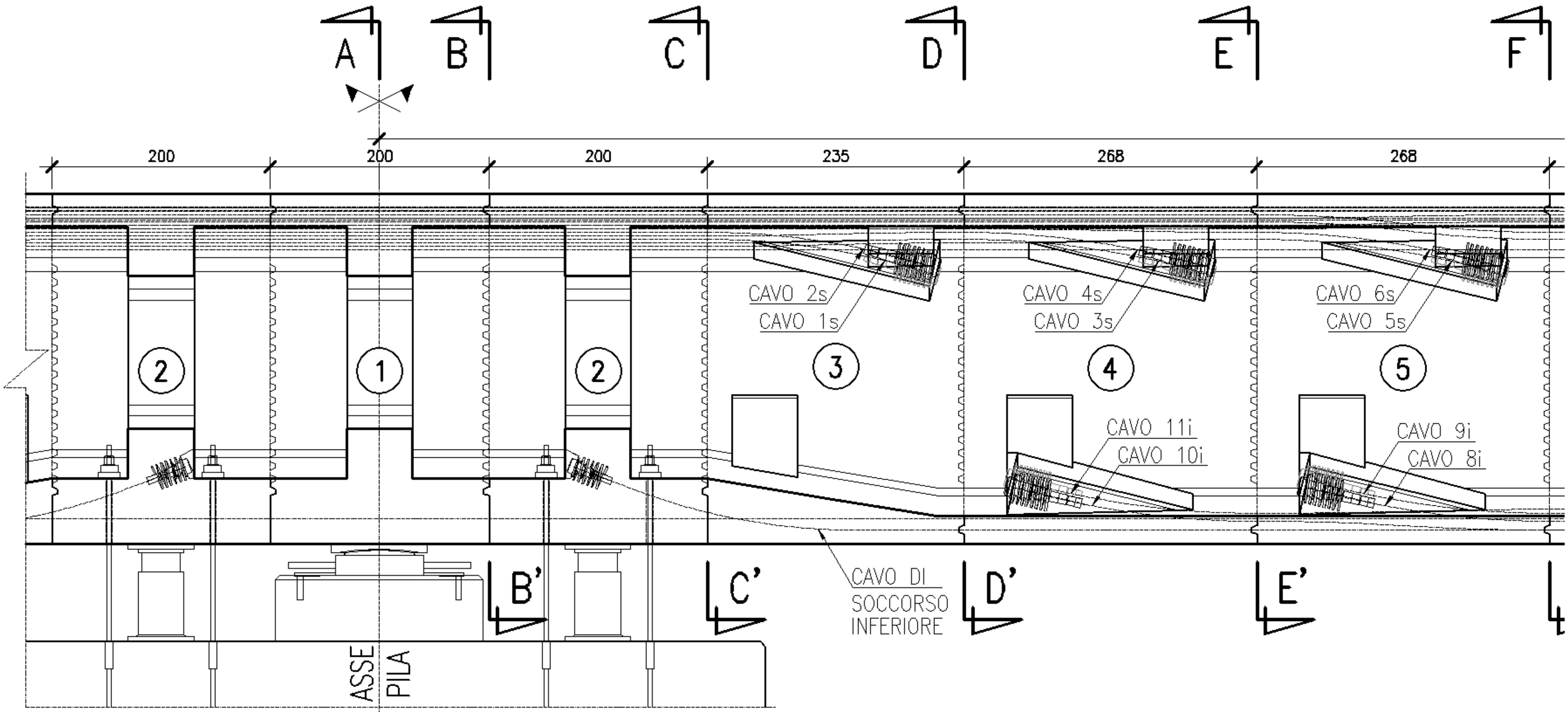
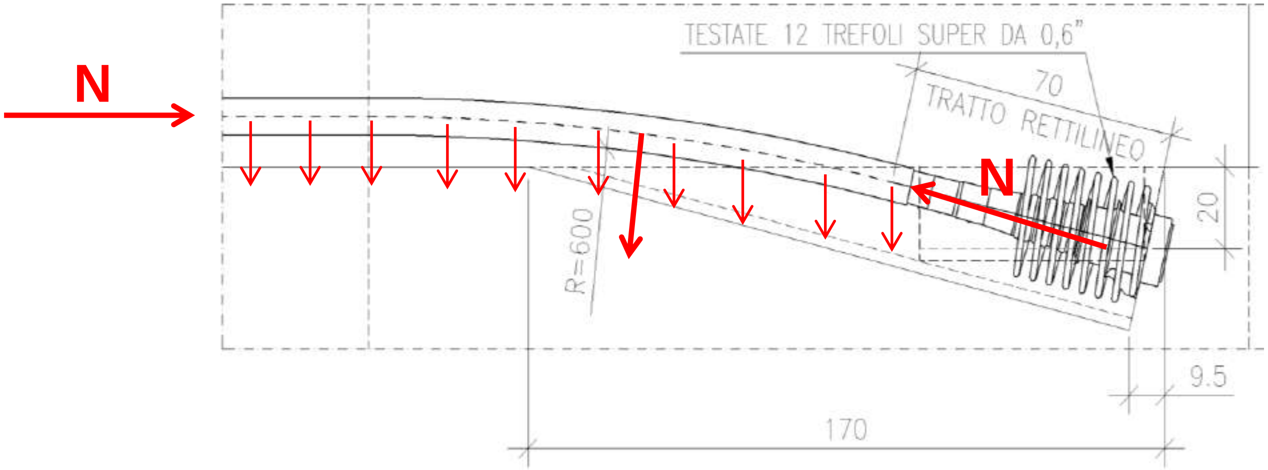


# CASE STUDIES: HIGHWAY BRIDGES

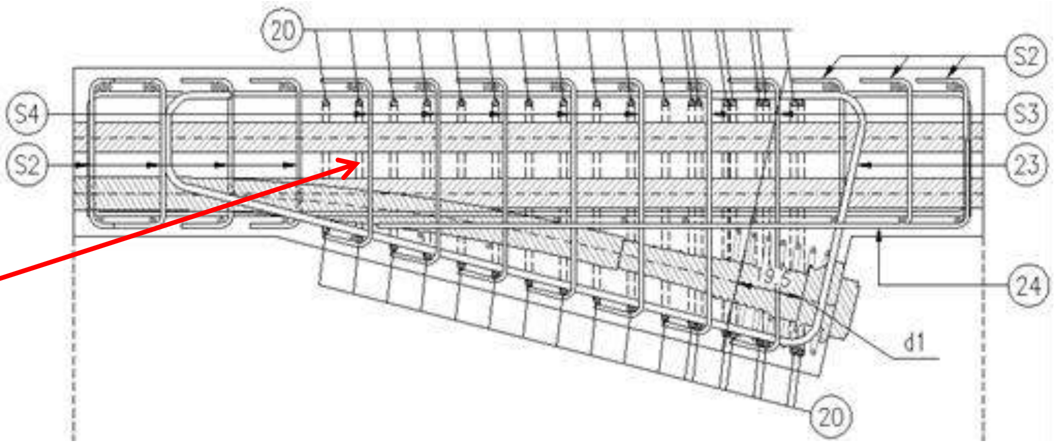
Damage observed on the precast element during the tensioning phase



# CASE STUDIES: HIGHWAY BRIDGES

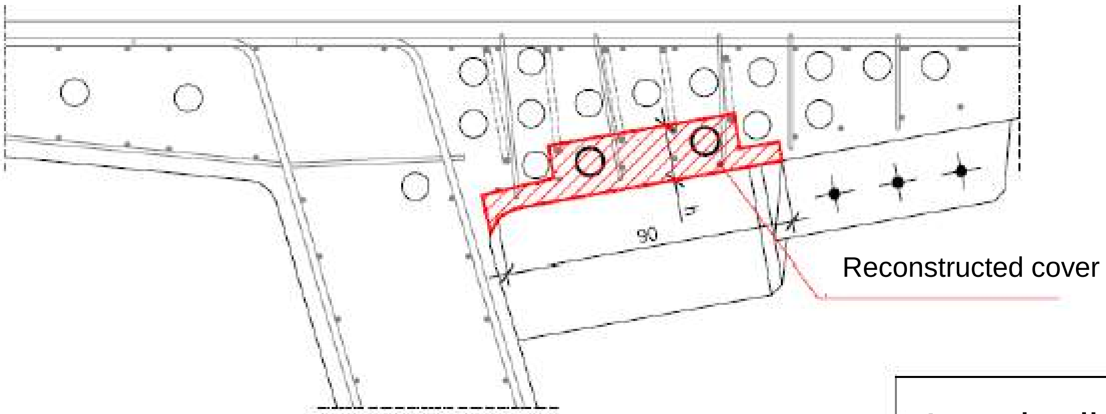
Damage observed on the precast element during the tensioning phase

Transverse rebar in the original executive drawings, there are no stirrups but only open reinforcement

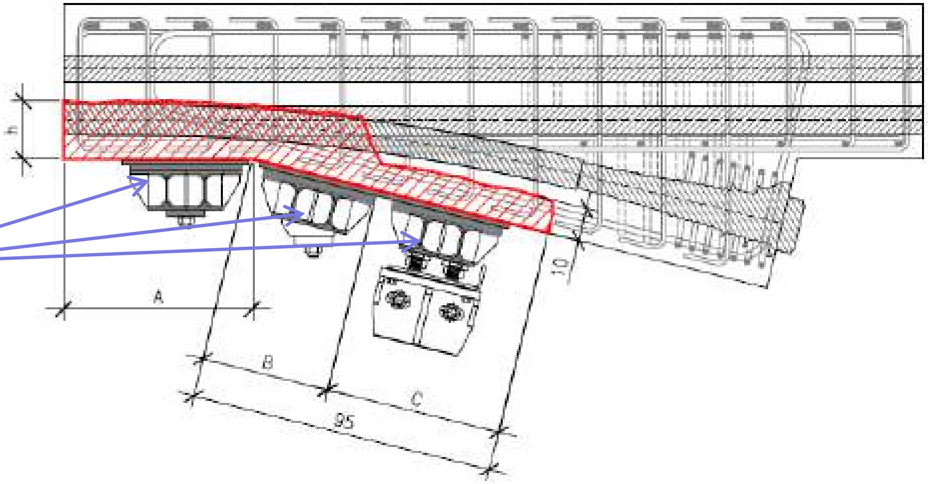


# CASE STUDIES: HIGHWAY BRIDGES

Trasversal Section




Longitudinal Section

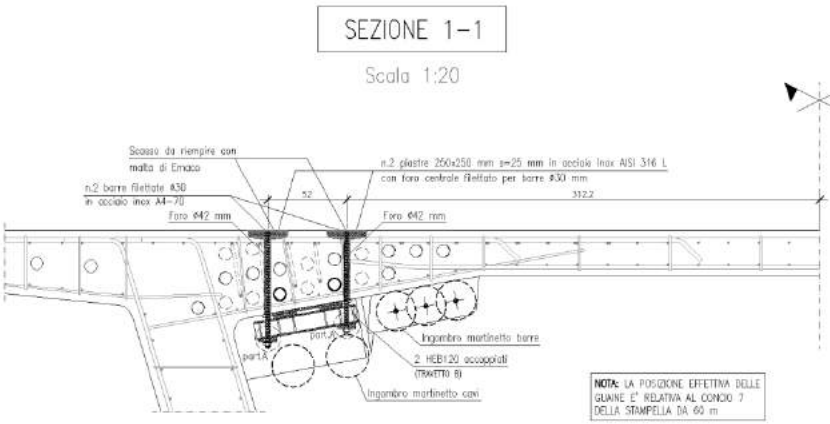


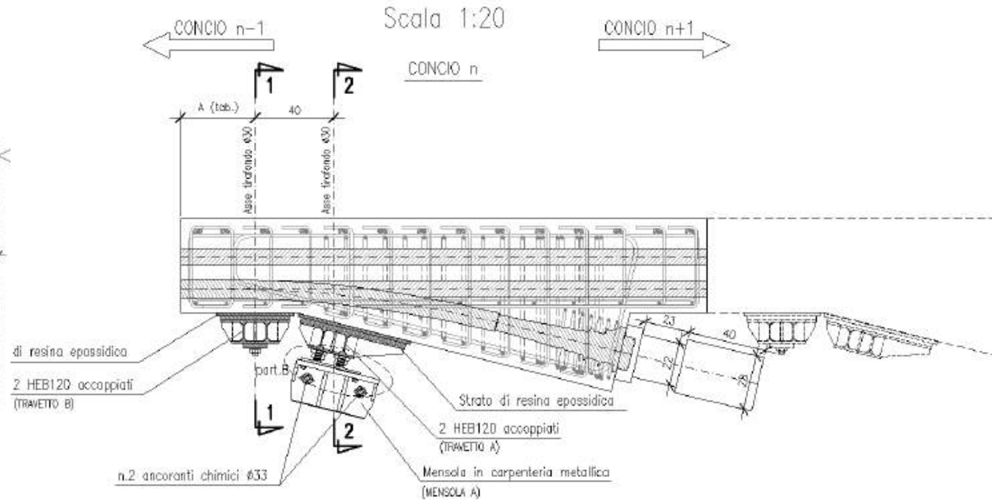

Strenghtening devices: steel profiles welded to steel plates, prestressed by introducing imposed relative elastic displacements between them and rigid restraints

## CASE STUDIES: HIGHWAY BRIDGES

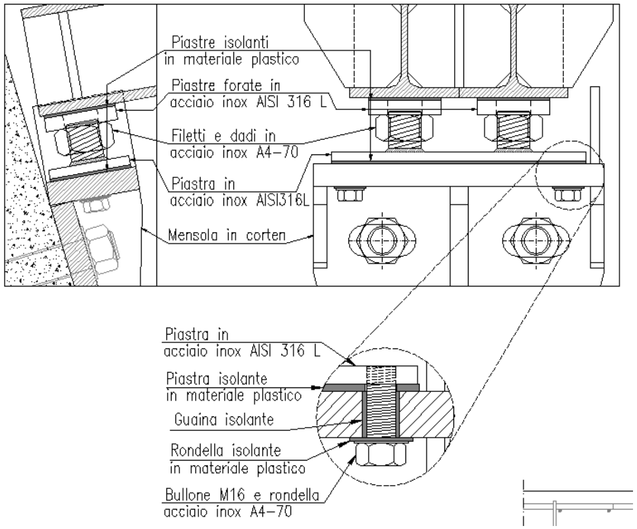
In the damaged segments the concrete was rebuilt by removing, using selective hydro-demolition techniques, alle the damaged parts to the ordinary reinfocement

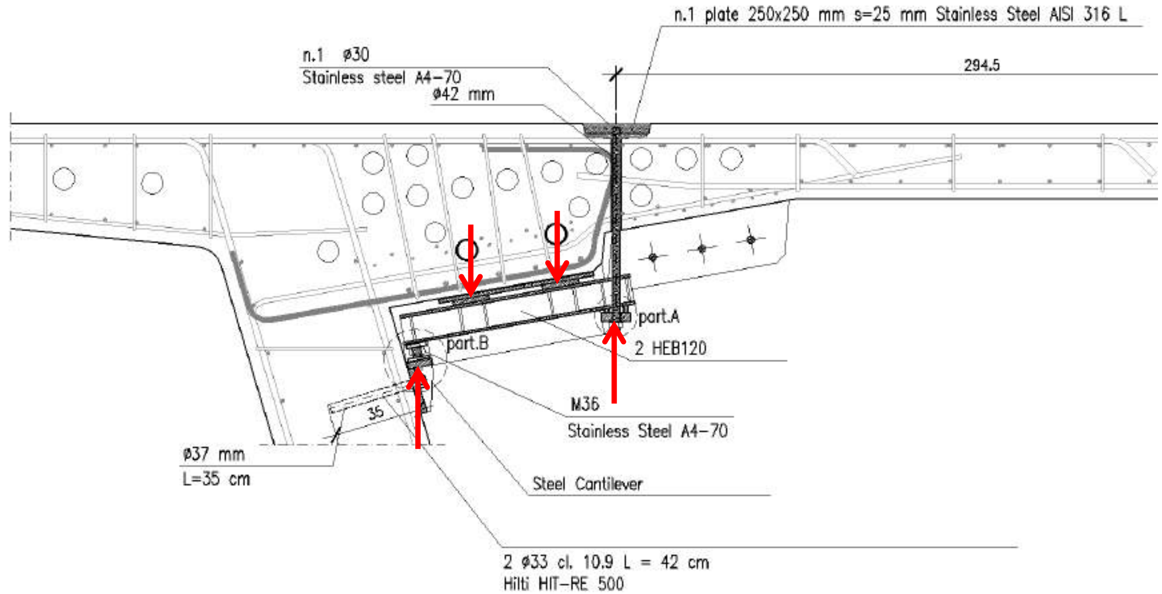
# CASE STUDIES: HIGHWAY BRIDGES



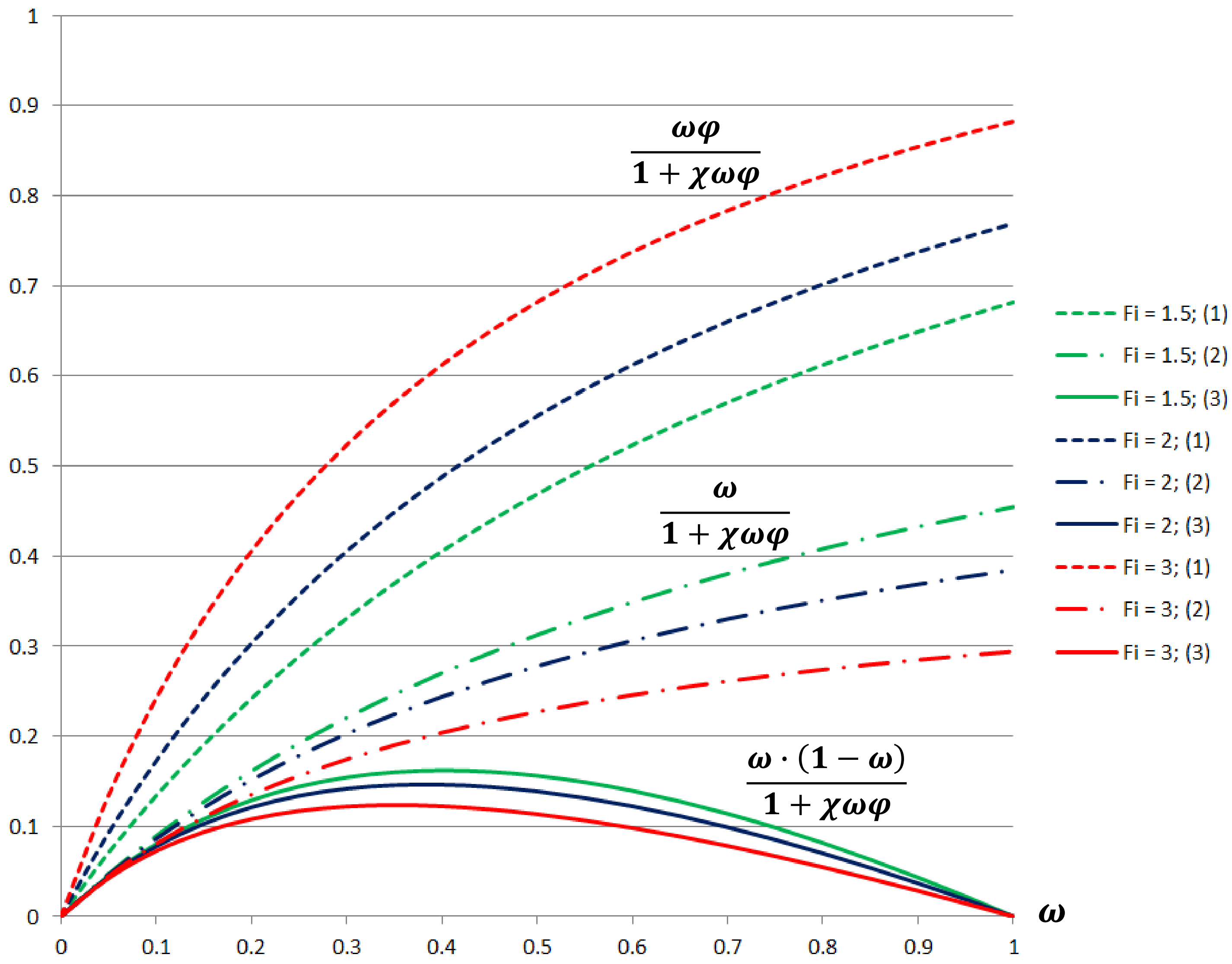




# CASE STUDIES: HIGHWAY BRIDGES

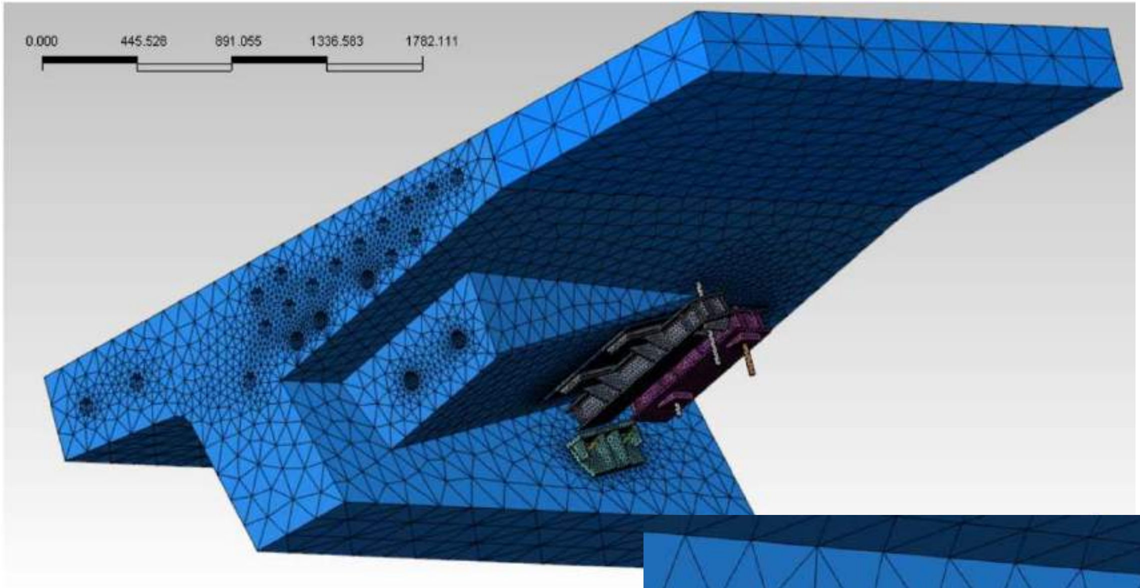


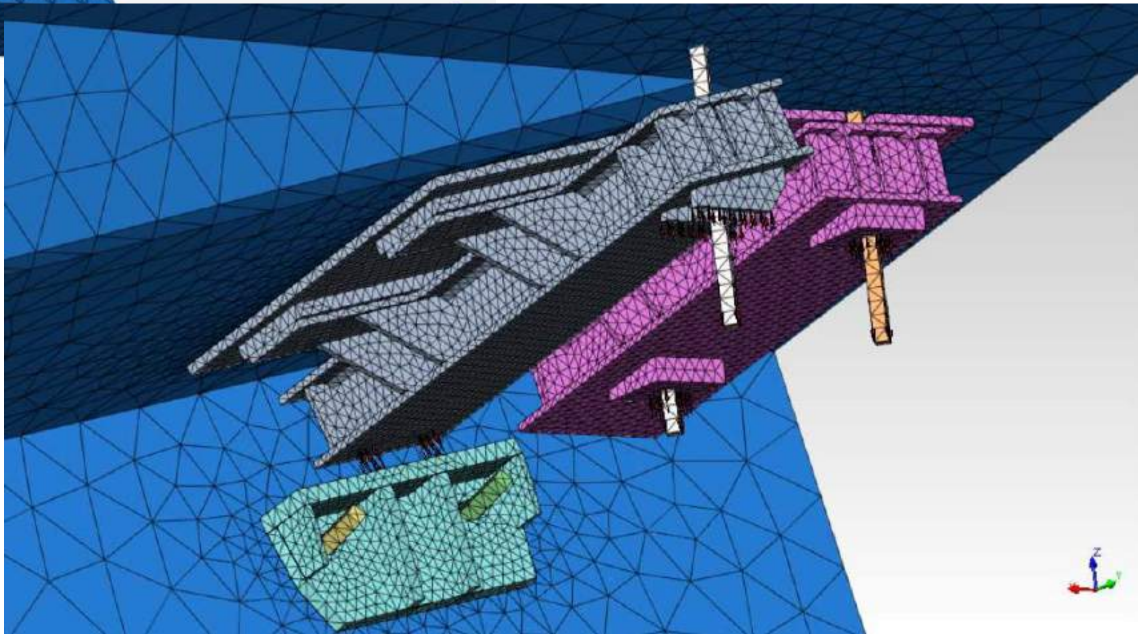




# CASE STUDIES: HIGHWAY BRIDGES

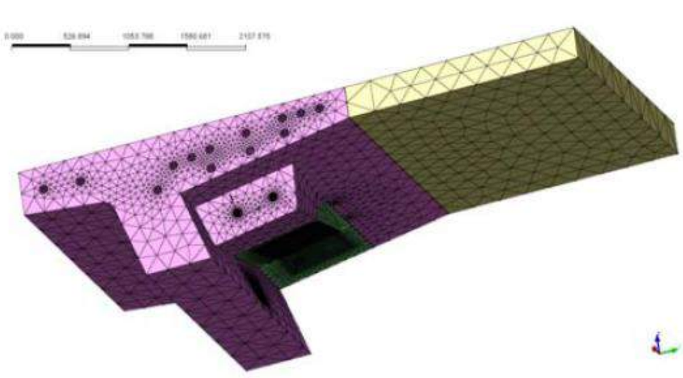


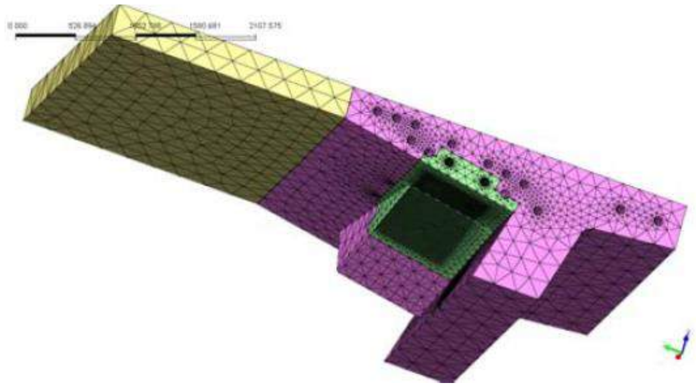
# CASE STUDIES: HIGHWAY BRIDGES



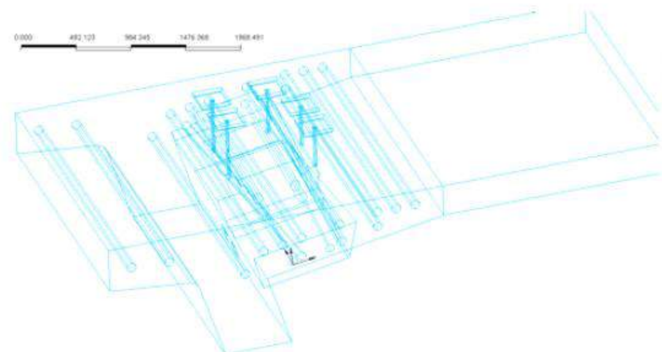
# CASE STUDIES: HIGHWAY BRIDGES

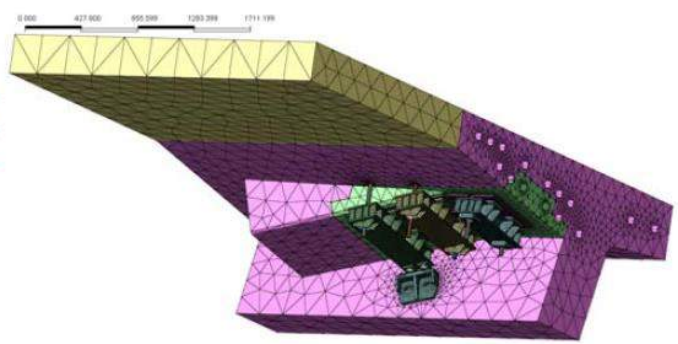


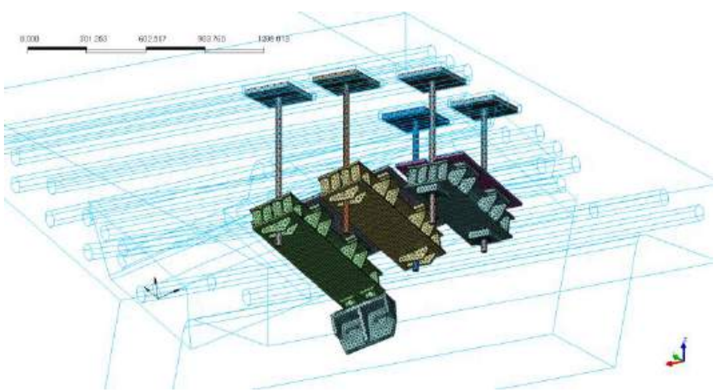


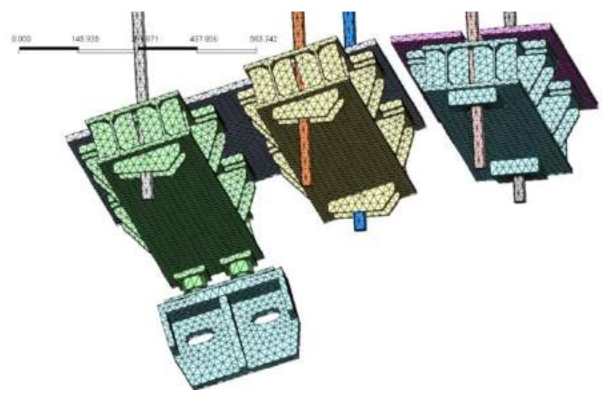
**Cross section geometry**

**Prestressing reinforcement**



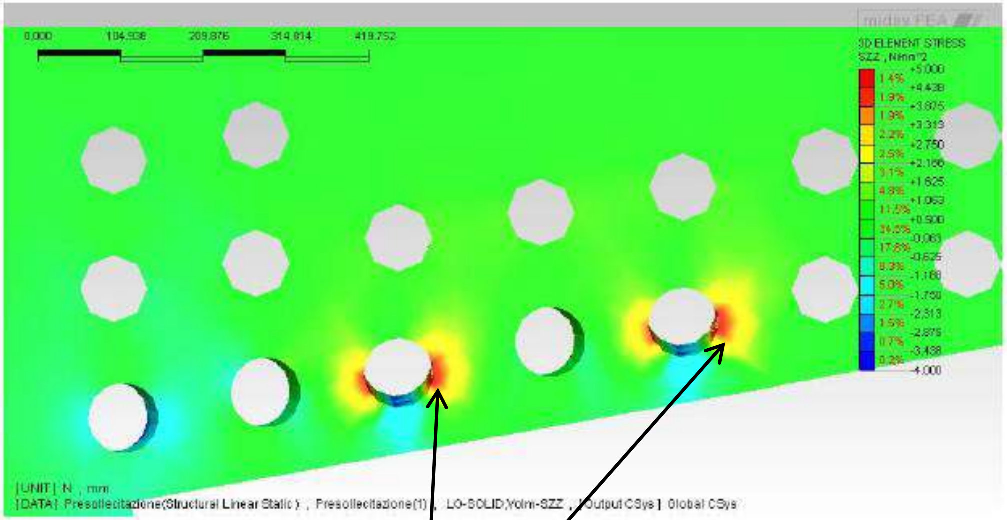






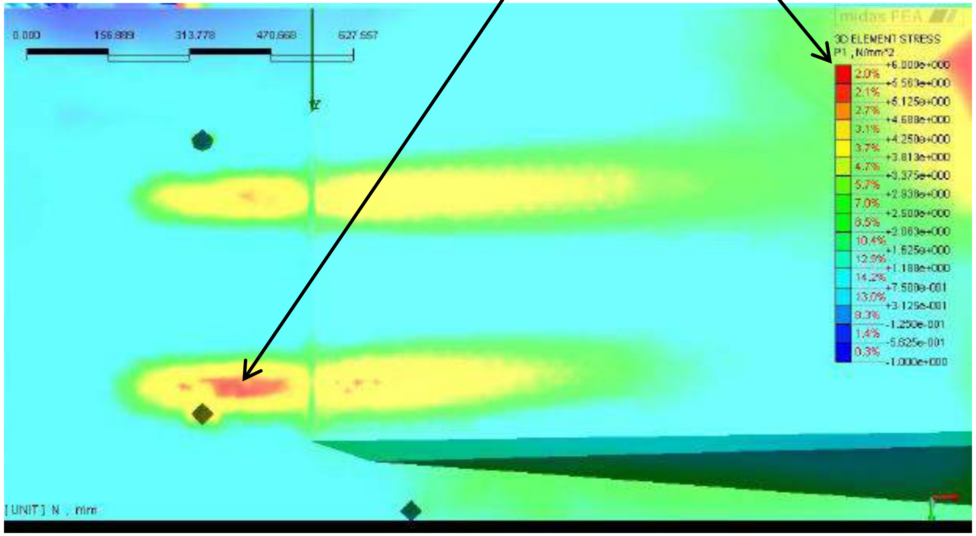
**Details**

# CASE STUDIES: HIGHWAY BRIDGES

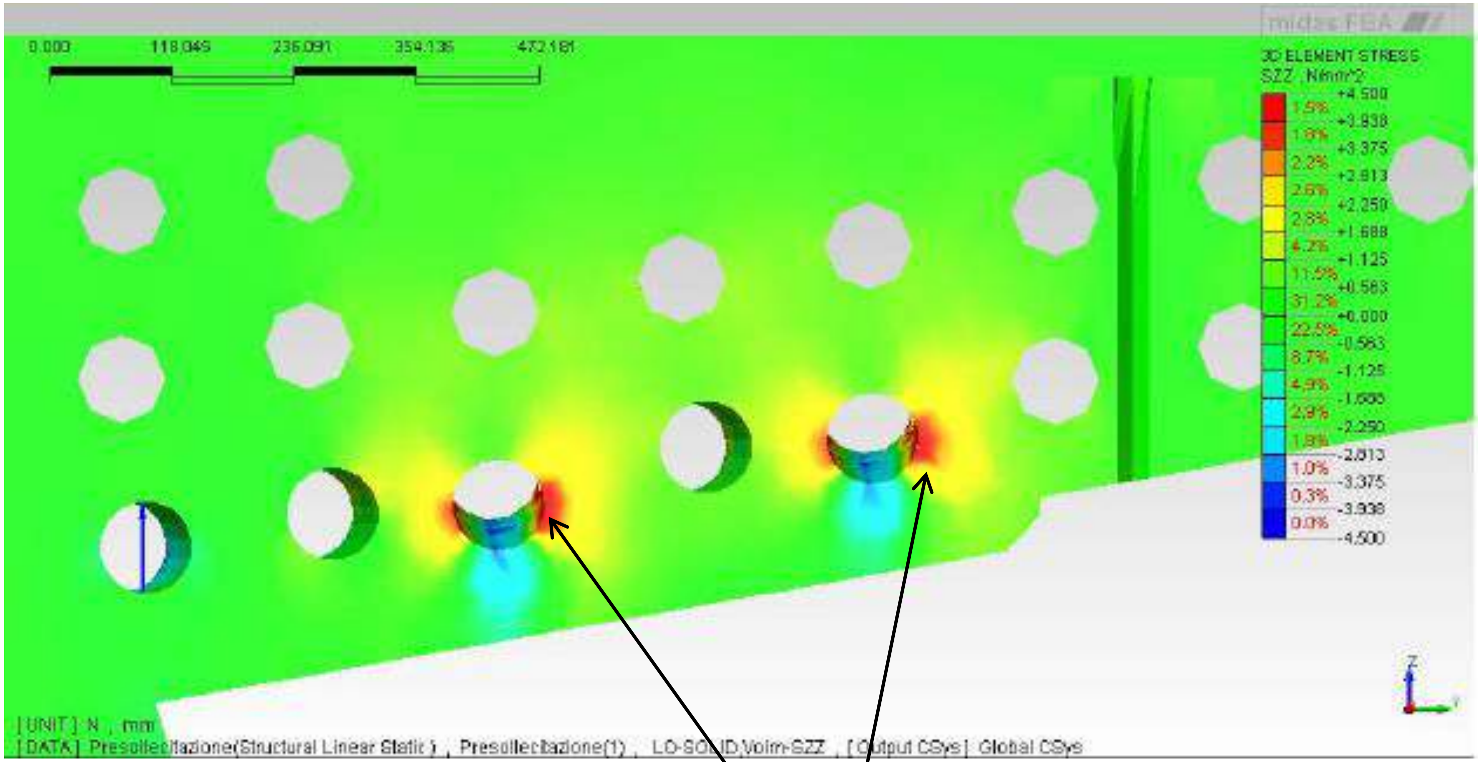


Principal stress of tension ≅ 6MPa

Vertical tension $\sigma_z \cong 5$MPa

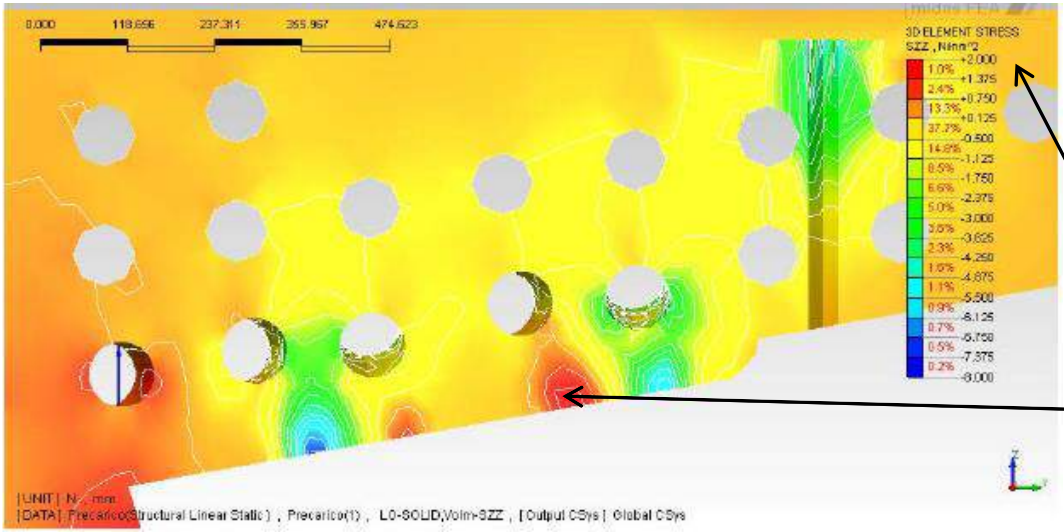


# CASE STUDIES: HIGHWAY BRIDGES




Vertical tension $\sigma_z \cong 4.5$MPa

# CASE STUDIES: HIGHWAY BRIDGES

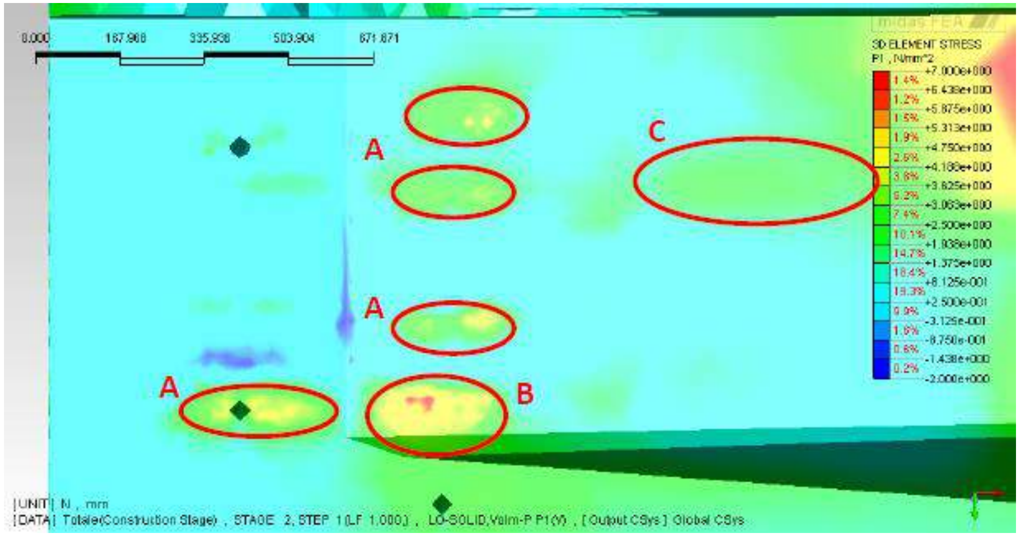



Introduction of a prestressing force about 50% of the total deviation force of the cable

Vertical tension $\sigma_z$ < 2MPa

Principal stress of tension:
- Zone A and B: local effect due to linear elastic model

- Zone C: tension due to prestressing less tha 2 MPa



# CASE STUDIES: HIGHWAY BRIDGES

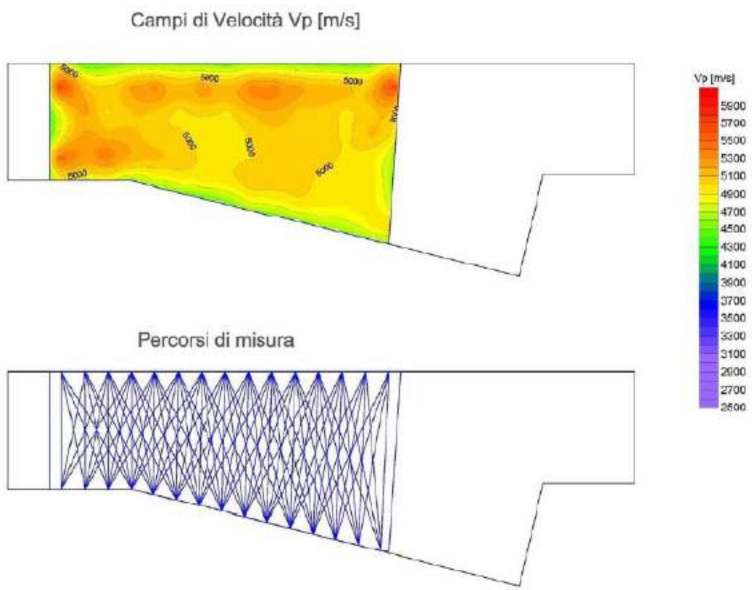



*Fig. 13 – Confronto tra rappresentazioni tomografiche eseguite su di un concio integro e su di un concio ove rilevati danneggiamenti dello strato corticale di calcestruzzo*

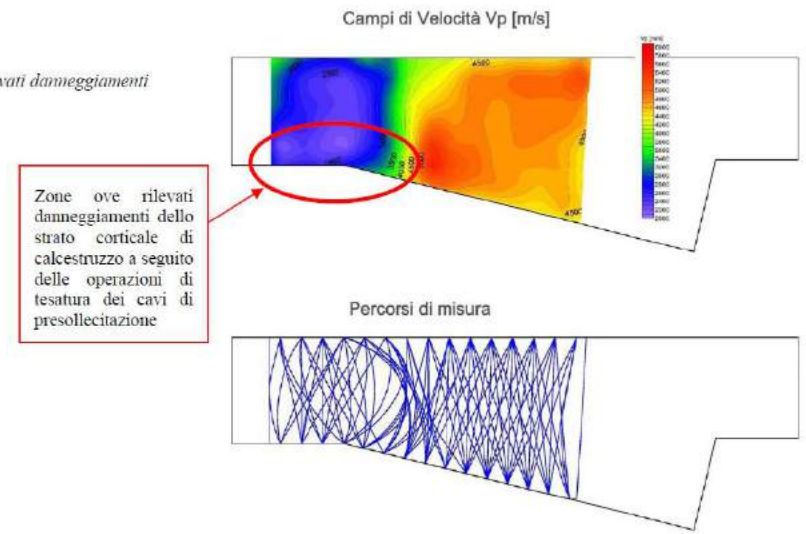
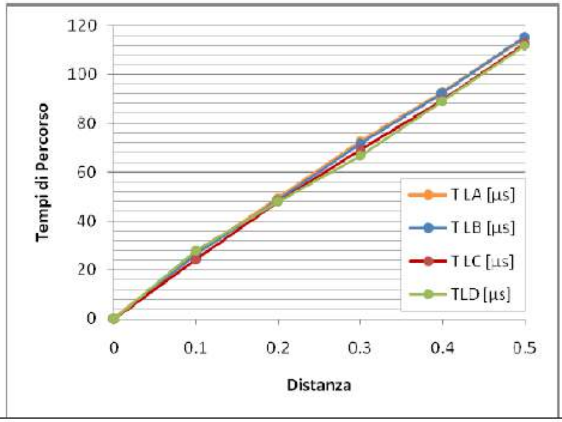


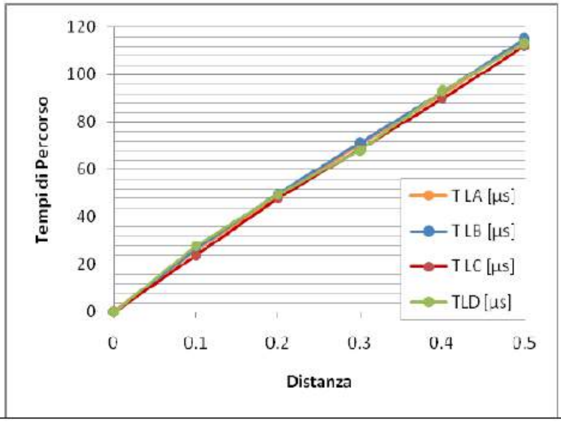

Figura 7: elaborazione tomografica della sezione A, concio 6A, pila P10N

*Fig. 14 – Rappresentazioni tomografiche eseguite su di un concio già varato (6A, pila 10N) ove rilevati danneggiamenti dello strato corticale inferiore di calcestruzzo*

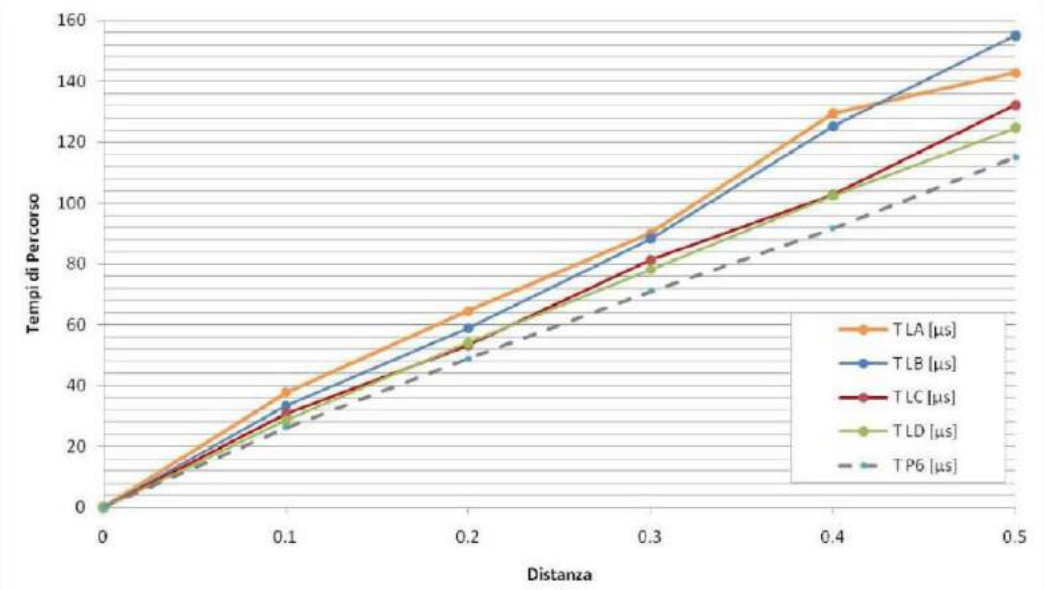
# CASE STUDIES: HIGHWAY BRIDGES



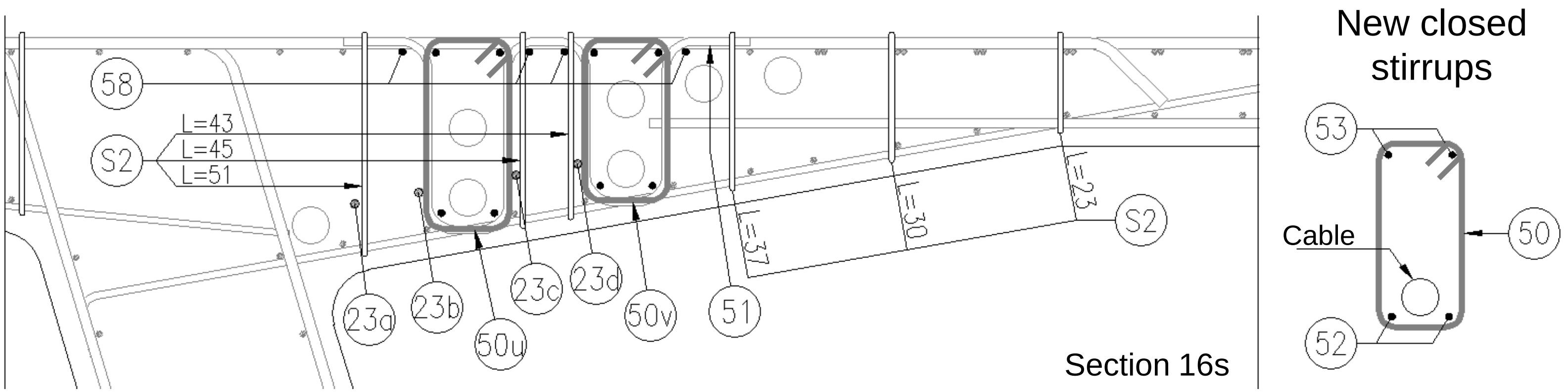

**Ultrasonic tests performed before prestressing the cables**




**Ultrasonic tests performed after prestressing the cables**



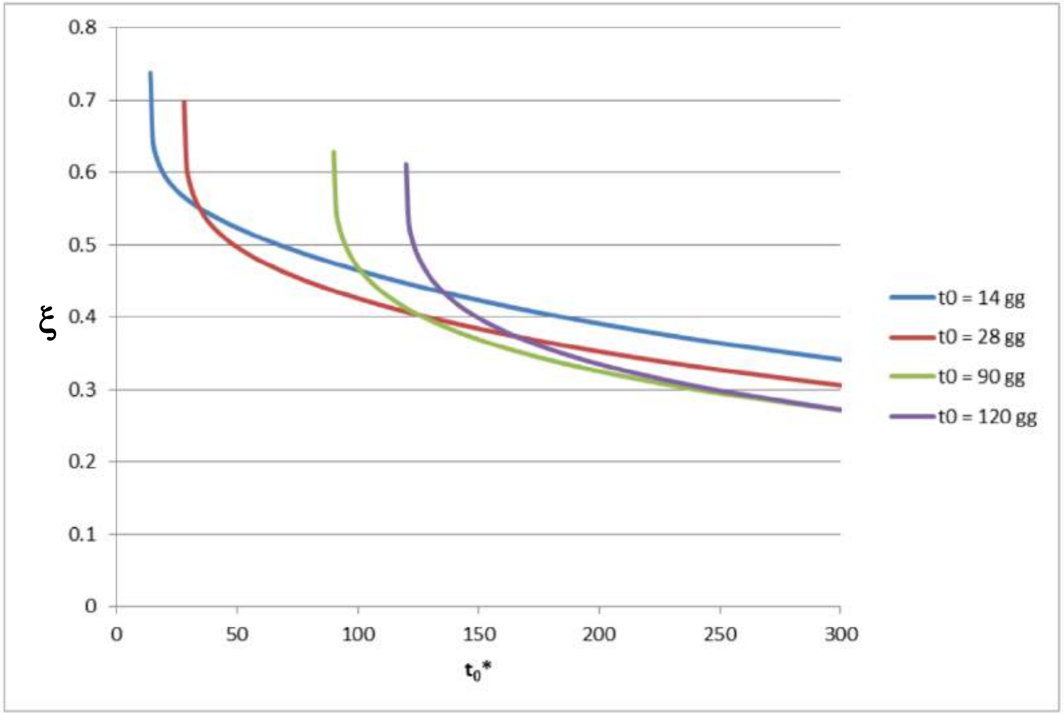
# CASE STUDIES: HIGHWAY BRIDGES



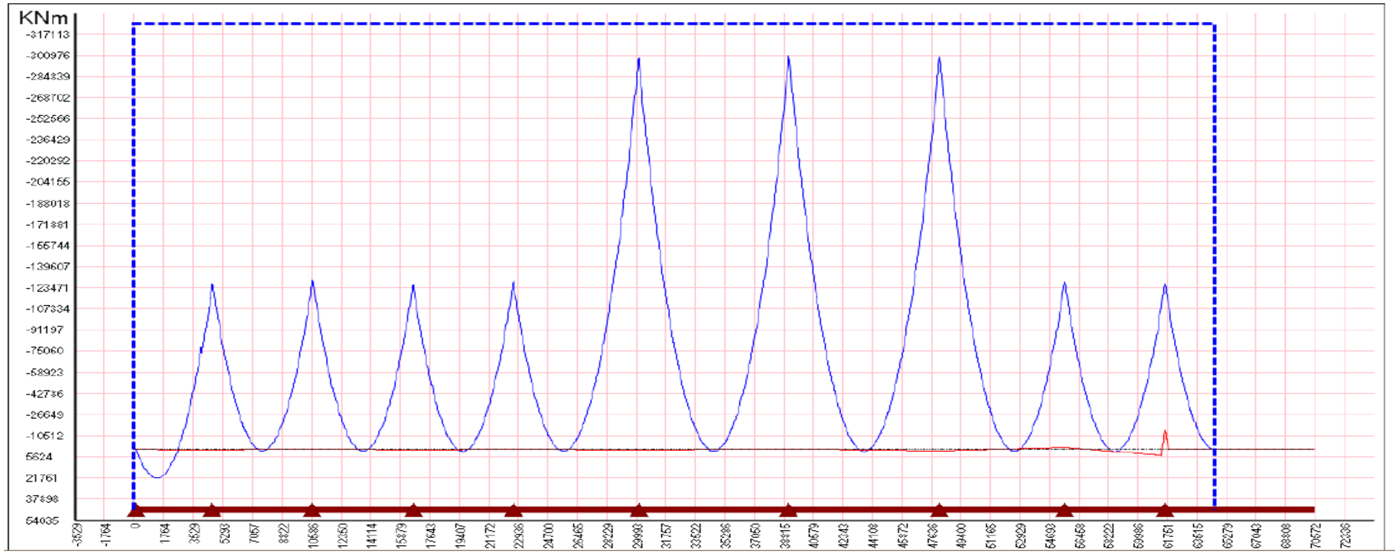

The equilibrium of the cables deviation forces has been guarateed with a more efficient distribution of stirrups with closed shape

# CASE STUDIES: HIGHWAY BRIDGES

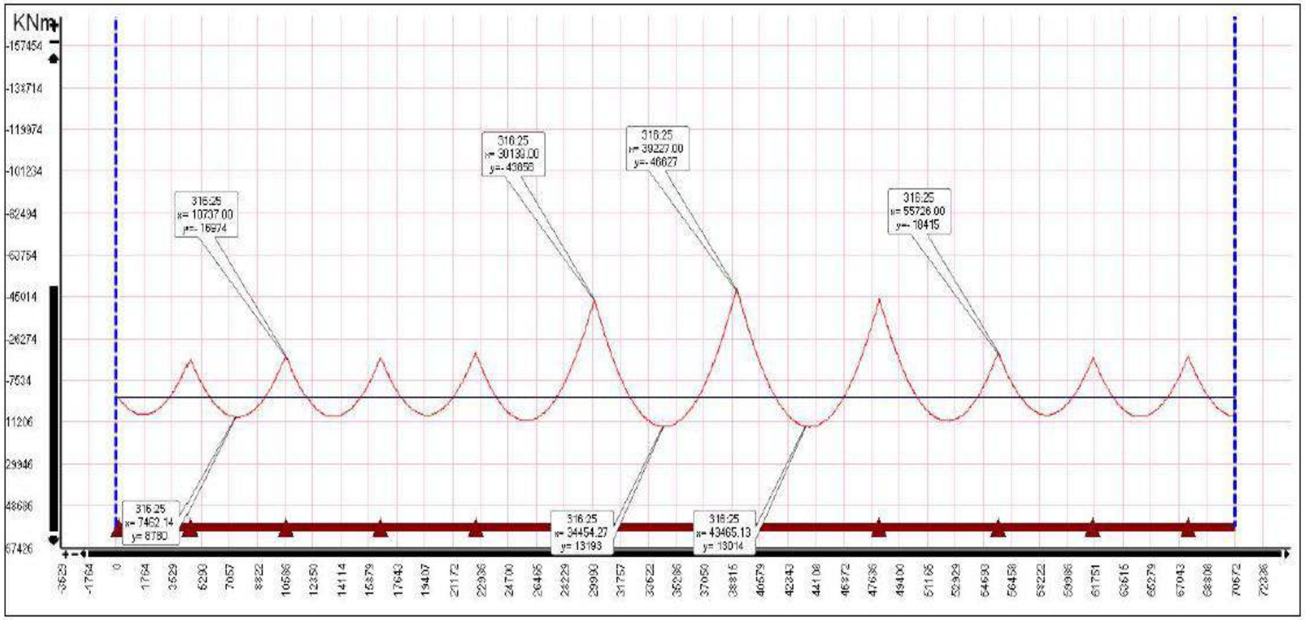
$$S(t) = S_0 \cdot (1 - \xi) + S_e \cdot \xi \qquad \xi(t, t_0, t_0^*) = \frac{[\varphi(t, t_0) - \varphi(t_0^*, t_0)]}{[1 + \chi\varphi(t, t_0^*)]}$$



# CASE STUDIES: HIGHWAY BRIDGES



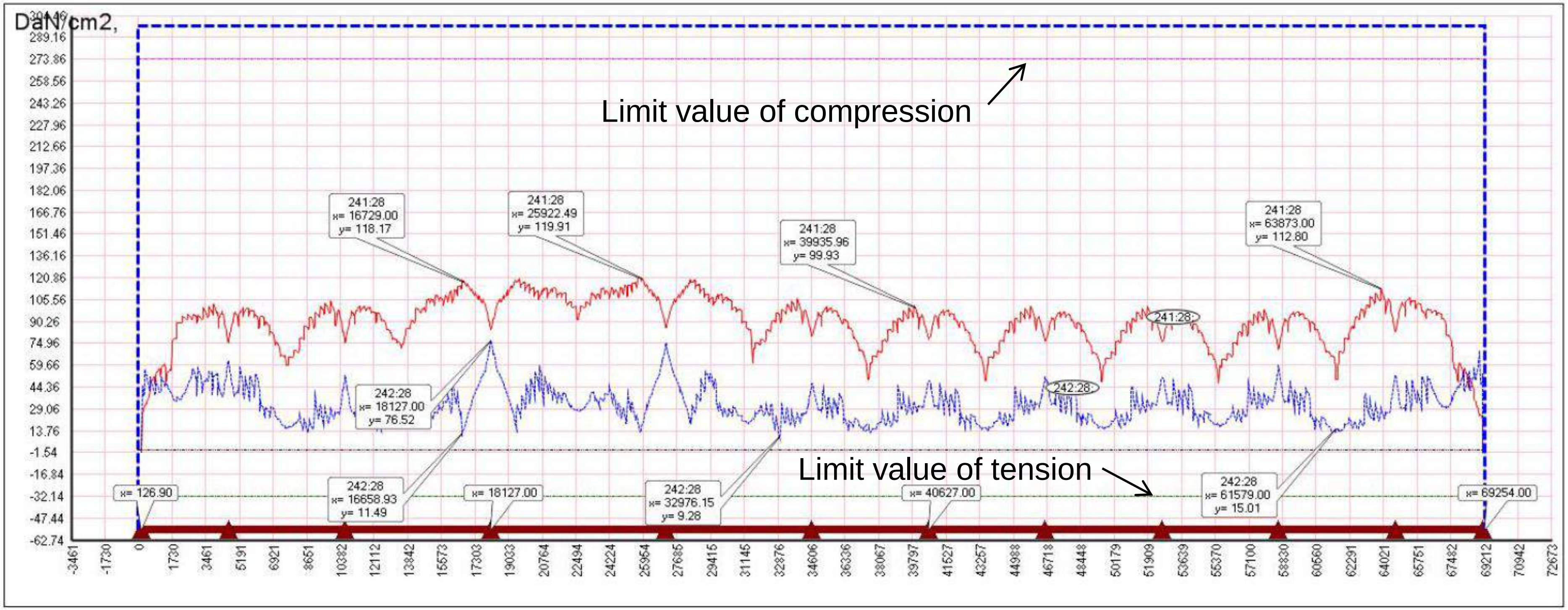

Distribution of bending moments in the configuration with maximum cantilever span




Distribution of bending moments in the final situation

# CASE STUDIES: HIGHWAY BRIDGES

Control of the stresses at the top and at the bottom of the section:



# THE BEGINNING OF A WONDERFUL HISTORY

**1920: JE SUIS UN ARTISAN**

**1930: UNE REVOLUTION DANS LA TECHNIQUE DU BETON**

**1938: UNE REVOLUTION DANS L'ART DE BATIR**

***"MY TIME WILL COME..."***

# “JE SUIS UN ARTISAN”

*E. Freyssinet*

*P. Sejournè*


$$v = v_e\,[1+k] \longrightarrow E' = \frac{E}{(1+k)}$$

**“LES DEFORMATIONS DU BETON SOUS CHARGES SOUTENUES SONT ISOMORPHES”**

$\bar{\varepsilon} = \varepsilon_{sh}$ **Deformation Imposeè (retrait)**

*Pont De Veurdre (1910), Vichy*

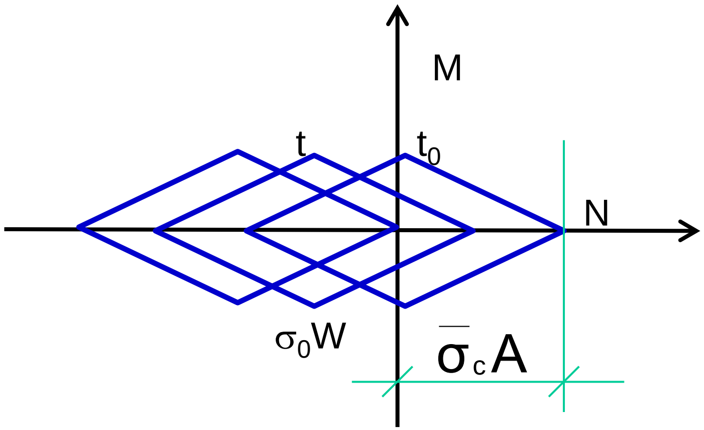
*Pont de Plougastel (1925-30), Brest*

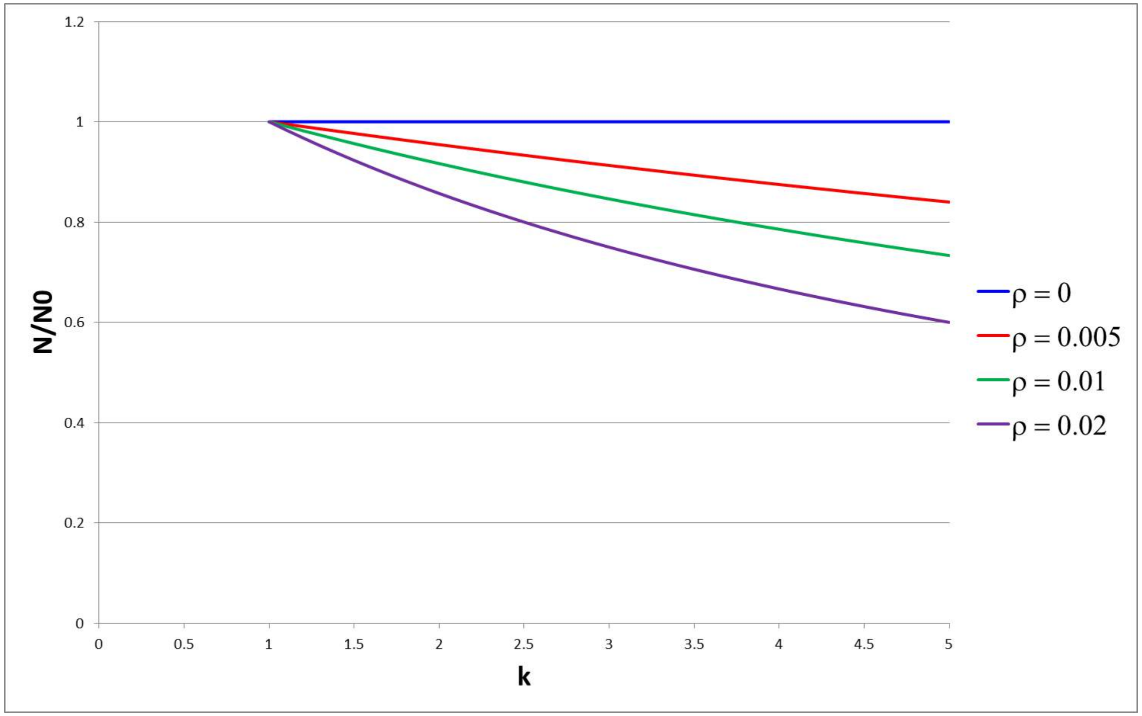
# THE ELASTIC MODULUS OF CONCRETE VARIES IN TIME

$$E_c(t) = \frac{E_c}{k} \qquad (k>1)$$

$$\frac{N}{N_0} = \frac{1+\alpha_e \rho_s}{1+k\alpha_e \rho_s} \qquad \alpha_e = \frac{E_s}{E_c}$$

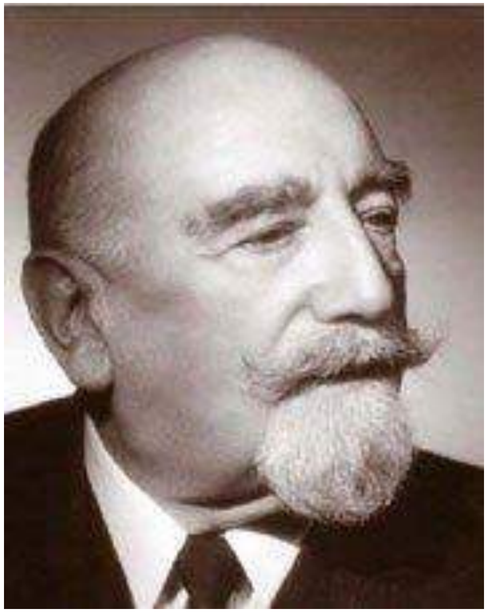
$$\rho_s = \frac{A_s}{A_c}$$



*The dark years*

A FORWARD-LOOKING CONTRACTOR



E. Campenon

1930

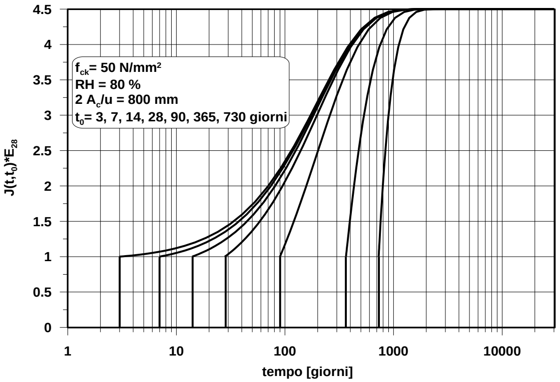
1933-39

## LINEAR VISCOELASTICITY AND EXTREME MODELS

### a) The Non-Ageing model and the asymptotical solution

$$\frac{\Delta\sigma_{s,N}}{\overline{\sigma}_s} = \frac{\alpha_e \rho_s (k-1)}{1 + k\alpha_e \rho_s}$$

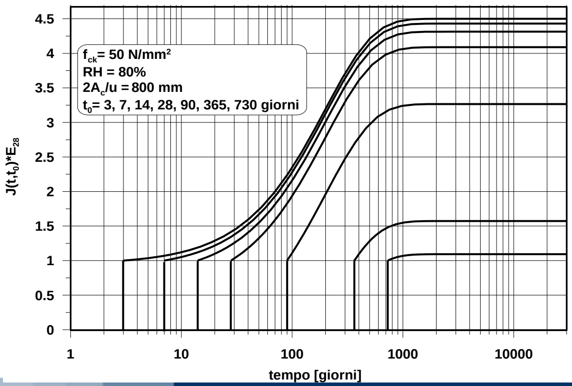
$$\Delta\sigma_{s,sh} = E_c \varepsilon_{sh\infty} \frac{\alpha_e}{1 + k\alpha_e \rho_s}$$




### b) The Dischinger Ageing model

$$\frac{\Delta\sigma_{s,N}}{\overline{\sigma}_s} = \left(1 - e^{-\frac{\alpha_e \rho_s (k-1)}{1 + k\alpha_e \rho_s}}\right)$$

$$\Delta\sigma_{s,sh} = \frac{E_c \varepsilon_{sh\infty}}{\rho_s (k-1)} \cdot \frac{\Delta\sigma_{s,N}}{\overline{\sigma}_s}$$



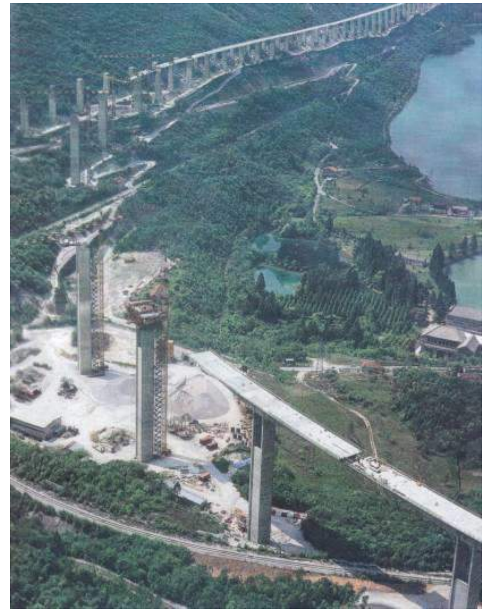
# THE MODERN FORMULATION OF MC HENRY

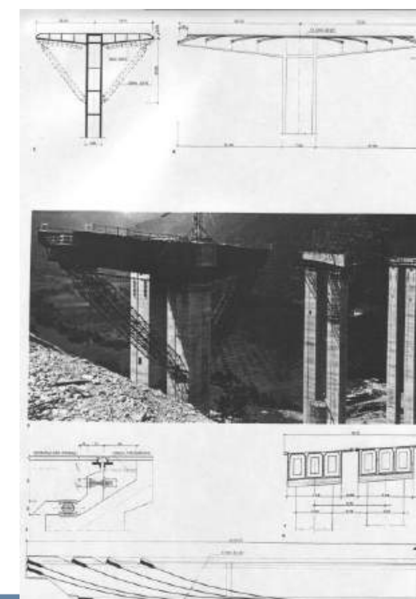
$$\varepsilon = \int_0^t d\sigma(t') J(t,t') + \varepsilon_{sh}$$

*Volterra and the theory of integral equation*

$$\sigma = \int_0^t d(\varepsilon - \varepsilon_{sh}) R(t,t')$$

# AFTER WWII AND THE TECHNOLOGICAL DEVELOPMENT





# THE ITALIAN CONTRIBUTION

Franco Levi

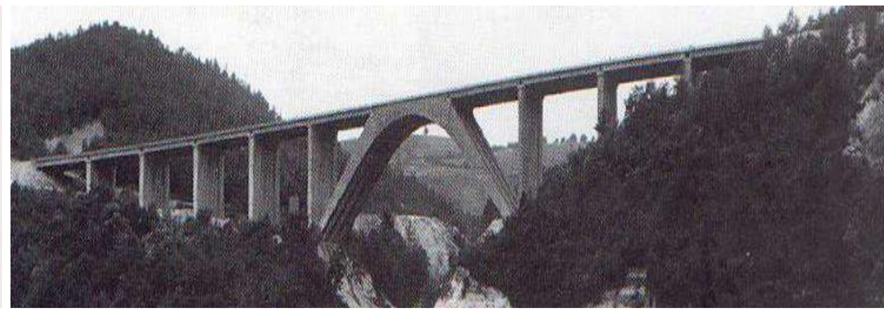
*Palavela, Torino*

*FIP Birth, 1952*

*CEB Birth, 1953*

*fib since 2000*

Carlo Cestelli Guidi



*San Giuliano Viaduct*

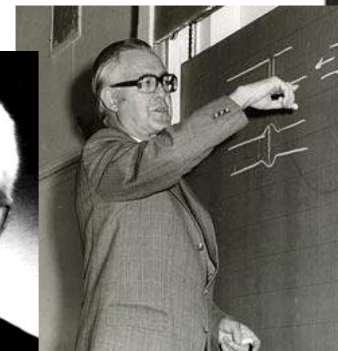
*Polcevera Bridge*

Riccardo Morandi

Silvano Zorzi

*Tagliamento Bridge*

# THE EUROPEAN DEVELOPMENT AFTER WWII



GERMANY

*Dickeroff*

*Rusch*

*Leonhardt*

*Finsterwalder*

*Campenon*

*Freyssinet*

*Guyon*

*Alimchandani (STUP- India)*

FRANCIA

# THE OPEN PROBLEMS IN PRESTRESSING

- The four verification classes
- The prestressing steel
- The anchorage devices
- The sectional optimization
- The ultimate capacity
- The shear resistance
- Limit State of Deformation
- The redundant structures
- The concordant cables and the theorems for continuous beams
- The great problem of the general solution of the delayed behaviour
- The protection against degrading and steel corrosion

# THE OPEN PROBLEMS IN PRESTRESSING

- Integral Solution
- Interaction between prestressing and normal steel
- Skew bending
- Special Effects (warping torsion)

# AT LAST: THE SOLUTION

*THE THEORETICAL FORMULATION*
*Systems of Volterra Integral Equations. The modal transformation of the unknowns. Canonical forms. The reduced relaxation functions.*

$$\sigma_{s,i} = \bar{\sigma}^{o}_{s,i} + E_s \underline{\rho}_i^T \underline{\underline{K}} \left[1 - \underline{\underline{\Omega}}\right]^{-1} \left[ \underline{\underline{I}} - \underline{\underline{\Omega}} \frac{\underline{\underline{R}}^*(t,t_0)}{E_c(t_0)} \right] \underline{\underline{K}}^{-1} \underline{\psi}_e$$

$$\underline{\underline{I}} - \underline{\underline{\Omega}} \frac{\underline{\underline{R}}^*(t,t_0)}{E_c(t_0)} = \left(\underline{\underline{I}} - \underline{\underline{\Omega}}\right) \int_0^t \frac{\partial \underline{\underline{R}}^*(t',t_0)}{\partial t'} J(t,t')\,dt'$$

$\underline{\underline{D}} = \left(\underline{\underline{B}}_c + \underline{\underline{B}}_s\right)^{-1} \underline{\underline{B}}_c$ = Coupling Matrix

$\underline{\underline{K}}$ = Modal Matrix

$\underline{\underline{\Omega}} = \left|\omega_{ii}\right|$ = Spectral Matrix

# AT LAST: THE SOLUTION

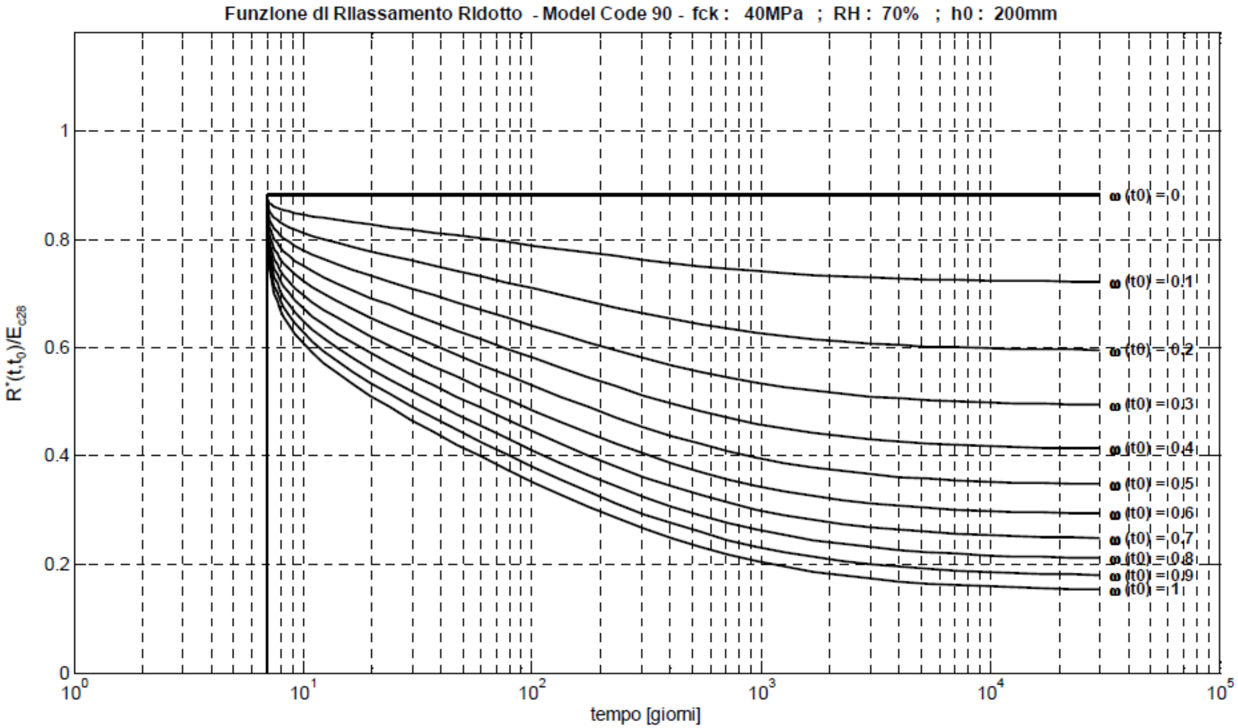
$$0 \leq \omega_{ii} = \frac{U_{c,i}}{U_{c,i} + U_{s,i}} \leq 1 = \frac{\underline{K}_i^T \underline{\underline{B}}_c \underline{K}}{\left(\underline{K}_i^T \underline{\underline{B}}_c \underline{K} + \underline{K}_i^T \underline{\underline{B}}_s \underline{K}\right)}$$

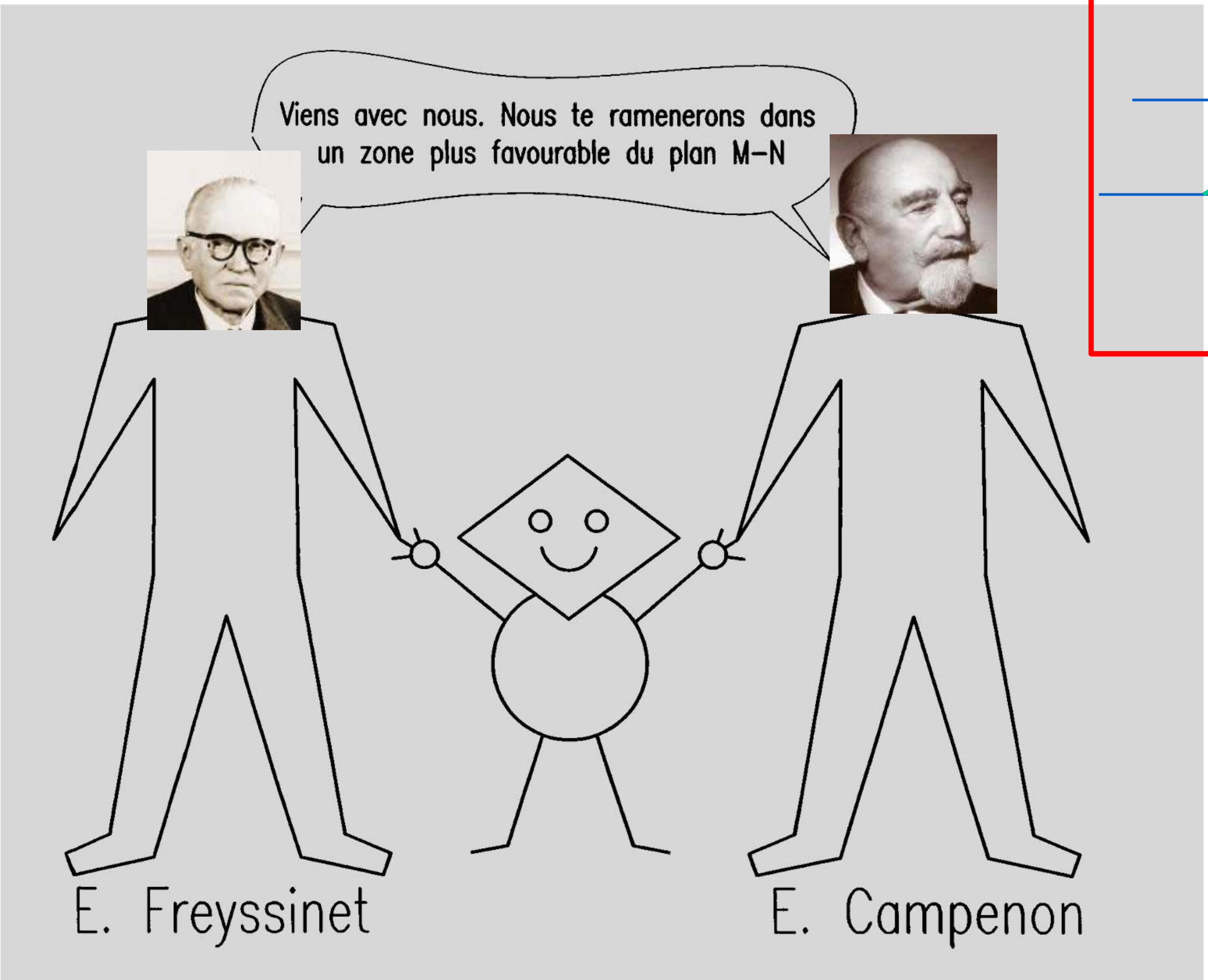
$$E(t_0) \cdot J^*_{ii}(t,t') = E(t_0) \cdot J(t,t') \cdot \omega_{ii} + 1 - \omega_{ii}$$

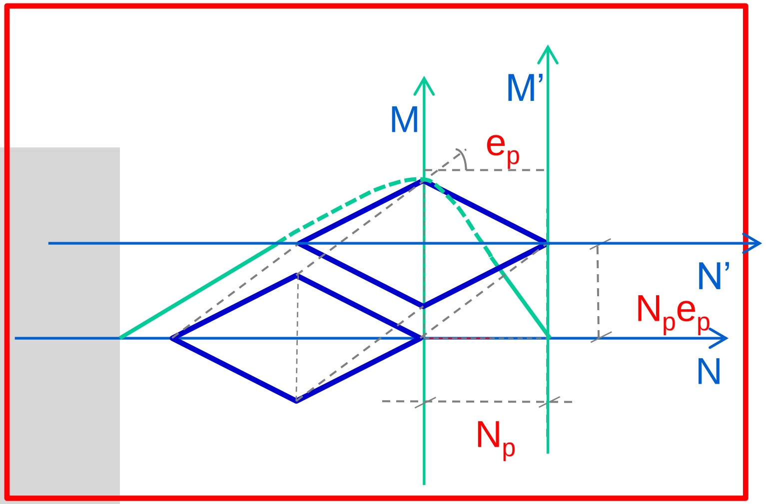
$$\int_0^t \frac{\partial R^*_{ii}(\tau,t')}{\partial \tau} \cdot J^*_{ii}(t,\tau) = 1$$

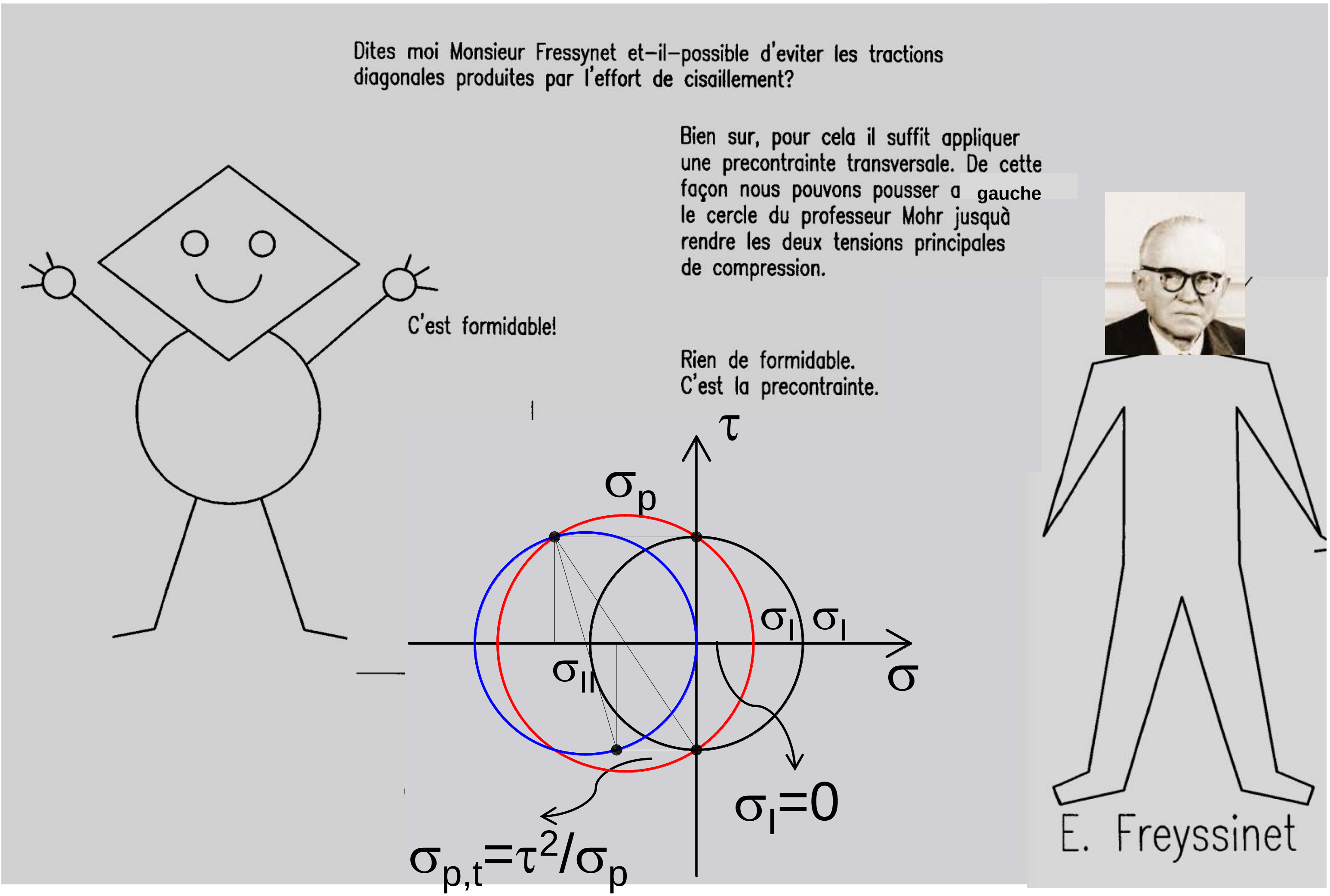
$J^*_{ii}$ = Varied Creep Functions

$R^*_{ii}$ = Reduced Relaxation Functions




Viens avec nous. Nous te ramenerons dans un zone plus favourable du plan M-N
E. Freyssinet
E. Campenon


M
M'
$e_p$
N'
$N_p e_p$
N
$N_p$


Dites moi Monsieur Fressynet et-il-possible d'eviter les tractions diagonales produites par l'effort de cisaillement?
Bien sur, pour cela il suffit appliquer une precontrainte transversale. De cette façon nous pouvons pousser a gauche le cercle du professeur Mohr jusquà rendre les deux tensions principales de compression.
C'est formidable!
Rien de formidable. C'est la precontrainte.
$\tau$
$\sigma_p$
$\sigma_I$ $\sigma_I$
$\sigma_{II}$
$\sigma$
$\sigma_I=0$
$\sigma_{p,t}=\tau^2/\sigma_p$
E. Freyssinet

# "MY TIME WILL COME … "

# 1888: A UNIQUE YEAR FOR SYMPHONIC MUSIC

**R. Strauss: Don Juan**

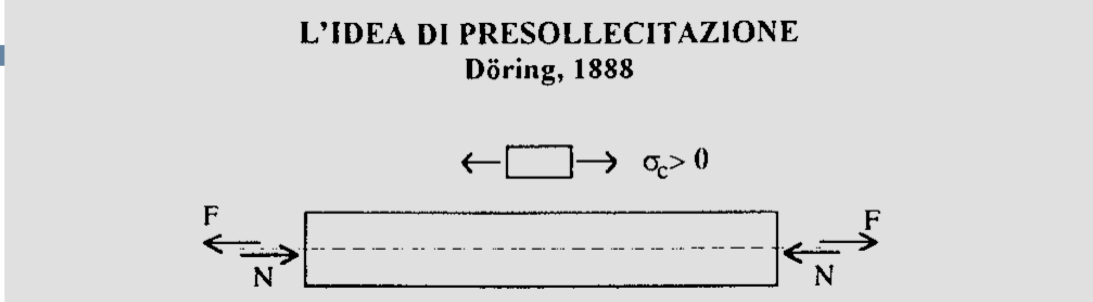
**N. Rimskij-Korsakov: Sherazade**

**A. Dvorak: VIII Symphony**

**P.Chaikovskij: IV Symphony**

**C. Frank: Symphony in D minor**

# 1889: A TITAN AND A DETESTABLE "ANDANTE"



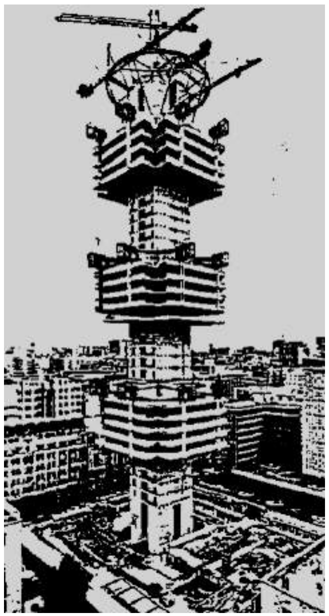


"Blumine" second movement of G. Mahler I Symphony in D major

# 1968: JOHANNESBURG AND NEW HAVEN: THE PEAK OF AN IDEA AND AN EXQUISITE “ANDANTE”

L’IDEA DI PRESOLLECITAZIONE

La Standard Bank di Johannesburg, 1970



MAHLER, I SINFONIA

2° rappresentazione nell’identica versione del 1889, dopo il ritrovamento del manoscritto di Blumine N. Haven, Aprile 1968


Frammento di “Blumine”

Così la Critica: “Blumine è un Andante raffinato ed è tempo che ritrovi il suo posto nell’affascinante scenario mahleriano”

Second performance in the same version of 1889, after the discovery of the long lost second movement “Blumine” N. Haven, 1968.

The critics: Blumine is an exquisite “Andante” and it is time will be restored in the fascinating mahlerian landscape with the posthorn of the III Symphony, the “Adagietto” of the Fifth and the amorous guitar of the Seventh…

# 1995: TOWARDS 300 METERS WHILE IN AMSTERDAM…

*Stolma Bridge*

*Gustav Mahler*

***"THE WORLD LISTENS"***